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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DETERIORATION OF CONCRETE DAMS DUE TO ALKALI-AGGREGATE REACTION

BY R. F. BLANKS [1] AND H. S. MEISSNER,[2] MEMBERS, AM. SOC. C. E.

SYNOPSIS

For many years engineers have been concerned over a peculiar and characteristic type of concrete deterioration, manifested by large-scale, random-pattern cracking. Such cracking is not unusual or new; in fact it is difficult to find massive concrete structures that do not display random cracking in some degree. Most instances of pronounced pattern or map cracking, as distinguished from ordinary small-scale crazing, have been attributed to excessive internal expansion, aggravated by varying degrees of surface drying shrinkage. The mechanisms producing internal expansion have been explained in the past on the basis of unsound aggregate or unsound cement, the latter resulting from high magnesia content or underburning and high free-lime content. Although some instances of expansive deterioration appear to be explained adequately by unsoundness, most cases, until recently, have remained enigmas and subjects for speculation only. Many concrete dams in all sections of the United States have developed evidence of this type of degeneration. Several of these cases have been caused by adverse chemical reactions between the alkalies in Portland cement and certain siliceous mineral constituents in the aggregates.

Identification of alkali-aggregate reactions as the cause of many cases of concrete deterioration has been recorded in contemporary technical literature. A brief selected bibliography is included as an Appendix of this paper. The destructive effects of the chemical instability of some aggregate constituents in the presence of alkalies contained in Portland cements have been definitely established. The problem is extremely complicated, however, and many questions, practical and technical, remain to be answered satisfactorily.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by June 1, 1945.

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Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams

The random-pattern cracking is very conspicuous, as may be observed from the photograph of a typical affected surface reproduced in Fig. 1. However, positive association of such surface manifestations with adverse alkali-aggregate reactions usually requires detailed investigation. Although crack

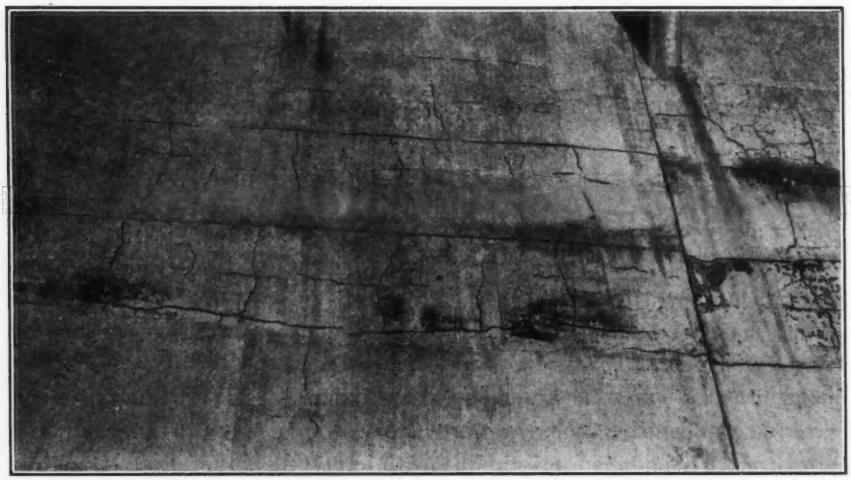

Fig. 1.—Expansive Reaction Between Aggregate and High-Alkali Cement Produces Typical Pattern Cracking on a Dam

openings of as much as 1.5 in. have been observed, they extend into the concrete only a comparatively short distance. The superficial nature of cracks caused by alkali-aggregate expansion has been established by drill cores, as illustrated in Fig. 2. The shallow depth to which such cracks penetrate has also been indicated by attempts to fill the openings by grouting. In one instance a hole was drilled parallel to a concrete face at a distance of 2 ft to intercept large, open cracks in evidence at the surface. Water under a pressure of 50 lb per sq in. failed to develop any connection between the surface cracks and the drill hole 2 ft inside the mass. Water tests from one surface to another, both displaying large, open cracks which from superficial evidence appeared to be connected, have been similarly unsuccessful.

Outward evidences of excessive internal expansion in concrete dams affected by adverse combinations of high-alkali cement and reactive aggregates are usually obscured by the massive proportions of the structure and the difficulty of establishing reference points for observing expansive movements. It is suspected that, under some conditions, restraint is sufficient to prevent most of the expansion that would normally occur if the concrete was not restrained. It is also suspected that such restraint of expansion tendencies in no way lessens the destructive action, except to reduce the surface crack openings.

In fact, deterioration may even be aggravated by internal stresses set up by the expansion forces. Some indication of the latter concept is given by the extremely low modulus of elasticity obtained in tests of cores drilled from affected structures.

Excessive expansion in massive concrete structures can sometimes be detected by the abnormal closing of expansion joints, particularly joints in a superstructure on a massive concrete foundation. Misalinement or displacement of hydraulic machinery such as the binding of gates, the cracking, bending, or buckling of penstocks, pit-liner plates, guide vanes, and speed rings, and the shifting or tilting of turbine-generator shafts are other signs of abnormal expansive movements in concrete which accompany alkali-aggregate reactions. Offsets at cracks, particularly in tunnels and galleries, frequently occur in affected concrete. Precise surveys and other forms of accurate measurements will sometimes indicate both the extent and direction of movement.

FIG. 2.—CORES REVEAL THAT WIDE CRACKS ARE OF SHALLOW DEPTH

Positive identification of the alkali-aggregate disease in concrete dams usually requires laboratory study and tests of cores drilled from the structure, and of specimens made with the particular aggregate used in the dam. Invariably, cores from affected concrete, when stored in the presence of moisture, will develop gelatinous exudations and whitish amorphous deposits on their surfaces. These gel deposits are also found in pores in the concrete mass, particularly adjacent to pieces of reactive aggregate. Chemical analyses of the gels show them to be composed essentially of sodium and potassium silicates. The gels may be transparent, closely resembling ordinary water glass as illustrated in Fig. 3, or they may be white or gray with a porcelaneous luster as shown in Fig. 4. Table 1 shows the results of chemical analyses of typical gel deposits from alkali-aggregate reactive concrete.

TABLE 1.—ANALYSES OF GEL DEPOSITS SHOW THEM TO BE COMPOSED ESSENTIALLY OF ALKALI SILICATES (VALUES ARE PERCENTAGES)

Type of gel sample	Loss at ignition	Silica SiO_2	R_2O_3	Lime CaO	Magnesia MgO	Sulfate SO_3	Soda Na_2O	Potassa K_2O
Bead	....	51.2	...	...	...	0	16.8	
Fig. 4	10.5	81.9	0.7	1.1	...	...	4.0	2.0
Fig. 3	20.5	53.4	1.5	4.7	0	0	12.7	4.5

Hand specimen and microscopic study of core samples from affected concrete also invariably reveal what appear to be zones of chemical reactivity, alteration, or infiltration around certain aggregate particles, as shown in Fig. 5. Aggregate pebbles which appear to have experienced reaction in concrete are

FIG. 4.—CARBONATED OPAQUE GEL ON A CONCRETE CORE

FIG. 3.—TRANSLUCENT GEL ON THE INTERIOR FRACTURED SURFACE OF A CORE

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FIG. 4.—CARBONATED OPAQUE GEL ON A CONCRETE CORE

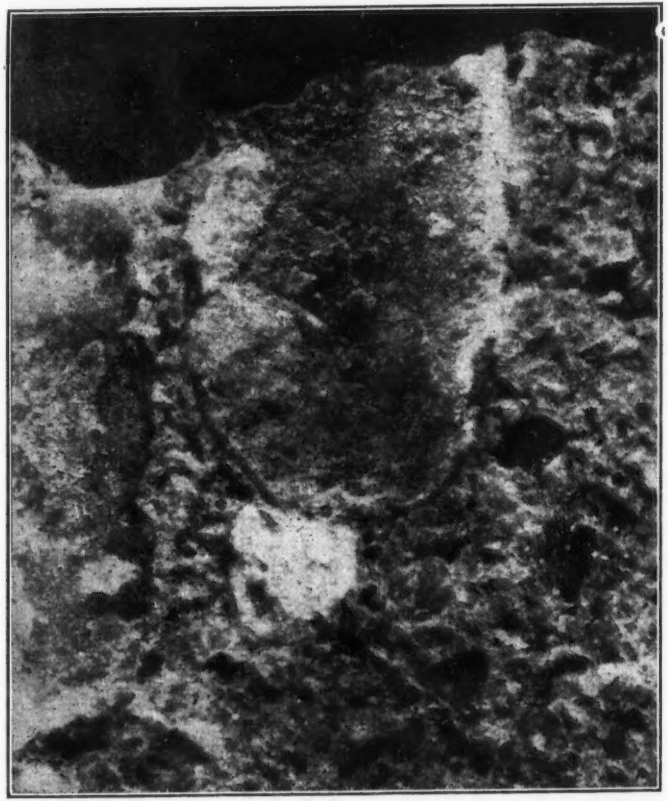

FIG. 5.—RIMS AND GEL POCKETS OFTEN SHOW ON REACTIVE AGGREGATE PARTICLES IN FRACTURED SECTIONS OF EXPANDING CONCRETE

FIG. 6.—CRACKS IN AFFECTED AGGREGATE PARTICLES EXTEND INTO THE PASTE AND ARE FILLED WITH GEL DEPOSITS

frequently softened and fractured. The cracks extend outward into the surrounding matrix and are usually wholly or partly filled with gel substances as illustrated in Fig. 6. The zones around reactive particles are characterized by discolored borders and loose sockets filled with gel material or other abnormal products.

Far too little is known about the specific mineral constituents of various rock types which contain silica in forms susceptible to reactivity with alkalies in cement to permit positive predetermination of troublesome combinations by petrographic analyses. Some of the more conspicuously reactive aggregate materials include: Opaline and other siliceous rocks such as some cherts, some siliceous limestones, some chalcedonies and intimate opal-chalcedony admixtures; certain acid to intermediate volcanic rocks such as some andesites and rhyolites, including tuffs and glasses of similar composition; and miscellaneous rock types such as phyllite. Eminent authorities have stated that all rocks, with the possible exception of pure limestone, are potentially reactive in concrete. Small amounts of adversely reactive substances are known to occur in a large number of aggregate materials; but it is not known, as yet, what is a tolerable limit below which amount such substances can be ignored and above which they must be considered as dangerous in combination with high-alkali cements.

In studying the various dams that are deteriorating because of alkali-aggregate reactions, it has been impossible in most cases to establish the exact alkali content of the cements used, because determinations for Na_2O and K_2O were normally not included in routine chemical analyses of Portland cements until quite recently. However, from information disclosed by occasional analyses made by the manufacturers, it has been proved with reasonable certainty that high alkali ($1.0 \pm \%$ $Na_2O + K_2O$) cements were used in all structures that have exhibited serious deterioration because of alkali-aggregate reaction. It has been possible to obtain samples of the aggregate materials actually used in constructing the various dams, which have been subjected to accelerated expansion tests in mortar or concrete bars. Again invariably, these aggregate materials have developed high expansions in comparatively short periods of time in combination with high-alkali cements and no abnormal expansion when used with low-alkali cements.

When correctly interpreted, results from laboratory expansion tests are in agreement with the service histories of a number of cases where reactive aggregates from a given source have been used in construction, with both high-alkali and low-alkali cements. The experiences from such cases have repeatedly demonstrated that concrete made with reactive aggregates and high-alkali cements develops expansive deterioration but the same aggregates in combination with cement of less than 0.6% alkali content cause little or no trouble. Typical examples of such service records are provided by the Parker, Gene Wash, and Copper Basin dams in comparison with the Gene and Intake pumping plants, all in southeastern California.

Parker, Gene Wash, and Copper Basin dams were completed late in 1937 or early in 1938. These structures were all built with the same aggregates

(a natural gravel deposit at the confluence of the Bill Williams and Colorado rivers in Arizona), and cement from the same mill containing 1.44% total alkalies. All three of these structures evidence profuse random-pattern cracking and expansive deterioration, which started from two to three years after completion. The Gene and Intake pumping plants were built almost coincidentally with the dams using aggregates from Colorado River deposits a few miles below the mouth of the Bill Williams River. Investigations have shown that the two aggregate sources (1) are of the same geologic age, (2) that they contain the same mineral elements in about the same proportions, and (3) that they show about the same degree of reactivity in the mortar-bar expansion test. The cement used in the pumping plants contained 0.51% total alkalies. After five or six years of service the concrete in these plants

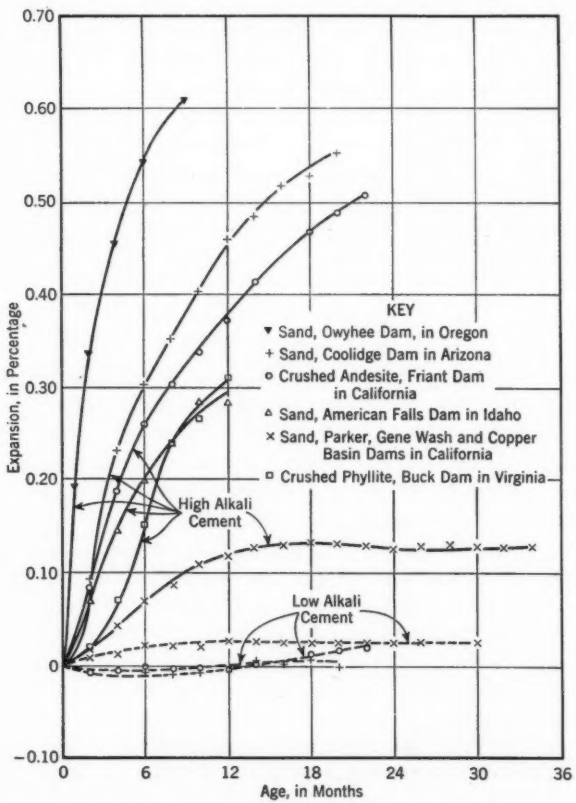

FIG. 7.—AGGREGATES USED IN SEVERAL DAMS EXPAND WHEN COMBINED WITH HIGH-ALKALI CEMENT

is as nearly perfect as any concrete could be. The same mill that supplied cement for the dams also later furnished cement for the Parker powerhouse but with a specification provision which limited the alkali content to 0.60% of ($Na_2O + 0.658\ K_2O$). The concrete in this latter structure, some of which

was poured the latter part of 1940, was still in excellent shape at the end of 1944, although the aggregate is known to give expansion with a high-alkali cement. Expansion-test data obtained with aggregates from various dams in combination with high-alkali and low-alkali cements are shown in Fig. 7. The mortar bars tested were first stored with moisture at 100° F, and sealed.

Although excellent correlations have been obtained between laboratory results and service performance, the mortar-bar expansion test has not been developed so that it can be relied upon as an infallible indication of troublesome combinations of cements and aggregates. Neither can it be considered as a satisfactory acceptance test for either cements or aggregates. The test is useful only as an indication of possible troublesome combinations of reactive aggregates and high-alkali cements. If high expansions are obtained in mortar bars within a period of three or four months, it may be concluded safely that the materials contained in the bars are reactive. However, slowly reactive aggregates may require much longer periods of time to develop excessive expansions in the bar test. In such cases it is not always easy to determine whether the expansion is due to inherent alkali-aggregate reaction tendencies or to extraneous causes. Furthermore, because of the highly accelerated nature of the test, excessive expansions may ultimately be indicated with a highly reactive aggregate and relatively low-alkali cements, although field records do not indicate that such combinations will necessarily cause trouble in actual service.

Progressive Development of Alkali-Aggregate Deterioration in Concrete Dams

The rate at which expansive reaction develops does not appear to follow any predictable schedule. Perhaps the variable nature of the progression is due to the fact that the action is influenced by so many factors, including the alkali content of the cement, the thickness of section, the quality of the concrete, and the conditions of exposure, particularly with respect to moisture conditions within and surrounding the concrete.

At Parker, Gene Wash, and Copper Basin dams, severe open-surface cracking developed with the appearance of warm weather two years after the structures were completed. The cracking extended rapidly until all exposed surfaces were covered by the random pattern. Measurements across typical cracks in these structures indicate that the cracks are increasing in width very slowly, if at all. Embedded instruments appear to record some slight expansive growth.

At Buck Dam in Virginia excessive expansion was apparently not observed until ten to fifteen years after construction. Surface cracking and expansion progressed until extensive repairs of the dam and power plant were required after some thirty years of service, as reported (1)[2] by Philip Sporn, M. Am. Soc. C. E., and H. A. Kammer, Assoc. M. Am. Soc. C. E.

Although the natural aggregate used in building Owyhee Dam in Oregon is very reactive according to expansion-bar tests, only two or three small

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

isolated areas of random-pattern cracking were in evidence in 1937, five years after its completion. The deterioration in these areas had progressed noticeably after another six years (1943), as shown in Fig. 8, and several other affected areas had appeared.

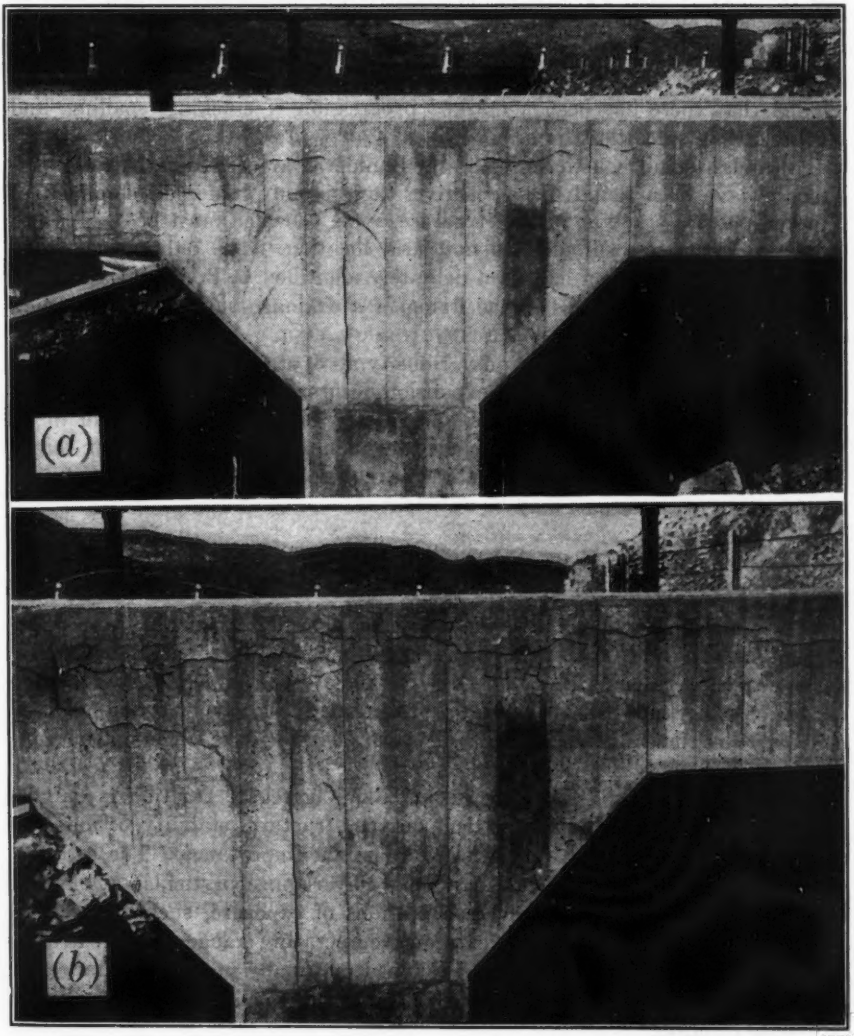

FIG. 8.—EXPANSIVE CRACKING HAS DEVELOPED PROGRESSIVELY IN THIS CONCRETE MEMBER, BUILT IN 1932

At Coolidge Dam in Arizona expansive deterioration has developed only in the parts where high-alkali cement was used, the greatest part of the structure (where low-alkali cement was used) being unaffected. During the period

of fifteen years since its completion, severe cracking has necessitated minor repairs to the parapets and maintenance work to correct binding of the reinforced concrete spillway gates.

American Falls Dam in Idaho was completed in 1925. In 1929 there was no evidence of random-pattern cracking. There was noted at that time an efflorescence of sodium carbonate on the walls of the gate-operating gallery. By 1940 all of the previously described symptoms of alkali-aggregate deterioration were in sharp evidence. From information available, it can only be concluded that there has been a marked progression of cracking, spalling, and disintegration since that time.

Although some constituents in the Boulder Dam (Arizona-Nevada) aggregate have been found to be reactive, there is only minor development of random-pattern cracking in the structure, some of which must be ascribed to extreme drying exposure in the hot, arid location. The only place where alkali-aggregate reaction had been definitely identified at Boulder Dam, after eight years of service, was in the lining of a drainage tunnel through some wet rock.

From information collected on various types of structures, it appears that, in those cases where early, rapid expansion occurs, the action generally seems to slow down or even stop after a few years. The slower cases seem to be more insidious and deterioration progresses, comparatively slowly, for thirty years or longer, or until rehabilitation is required. Apparently almost any combination of conditions may be anticipated.

Significant Effects of Alkali-Aggregate Deterioration on the Service Life of Concrete Dams and Appurtenant Works

The preceding evidence has been given in considerable detail in order to present as clearly as practicable the symptoms by which alkali-aggregate degeneration is recognized, how it may be diagnosed positively, and a brief review of some known case histories. The next question of more immediate concern to engineers is the manner in which the "disease" affects structures in service.

Since the cracking that accompanies alkali-aggregate reaction is only a superficial surface condition, its effect on the structural stability of massive concrete structures would appear to be of minor importance. The esthetic and psychological effects of such cracking may require particular attention. Concrete dams subjected to severe conditions of exposure, such as freezing and thawing, corrosive waters, extreme temperature ranges, and extreme drying shrinkage, of course, will be weakened materially in their resistance to the disintegrating elements by the surface cracking which accompanies alkali-aggregate reaction.

Concrete cores drilled from affected structures show that the interior concrete is characterized by a chalky, lifeless appearance. Cores break easily during drilling and it is difficult to obtain representative samples sufficiently intact for strength and elasticity tests. Therefore, strength tests with cores from alkali-aggregate concrete must be considered as representing the best of

such concrete. In general, such tests reflect some retrogression in strength as compared with that which might be expected from sound concrete under comparable conditions. In most cases, however, core samples have developed higher strengths than might be expected from their dead appearance.

Cores taken from Parker Dam, three years after construction, averaged 3,480 lb per sq. in. in compressive strength, with an average modulus of elasticity of 2,200,000. Previous cores, six months old, gave an average strength of 4,180 lb per sq in. and an elastic modulus of 3,800,000. In contrast to these results, 28-day control cylinders made during construction had an average strength of 4,180 lb per sq in., and 1-yr laboratory tests on Parker Dam concrete yielded 4,620 lb per sq in. strength and showed an elastic modulus of 4,200,000.

Similarly, although 28-day control cylinders showed the concrete in Gene Wash and Copper Basin dams to produce 3,450 lb per sq in. compressive strength, cores taken two years later showed the strength of this concrete to be 3,295 lb per sq. in., and the elastic modulus to be 2,420,000. Two-year-old cores, taken from pumping-plant structures which were built with the same aggregate as that used on Gene Wash and Copper Basin dams, but with low-alkali cement, were found to give 5,775 lb per sq in. compressive strength and to have a modulus of 4,800,000.

Mr. Kammer, with R. W. Carlson, Assoc. M. Am. Soc. C. E., reports (2) the strength of cores from the 26-yr-old concrete in Buck Dam as 2,100 lb per sq in.; and cores from the 15-yr-old affected concrete in American Falls Dam gave 2,840 lb per sq in. The latter concrete showed a low elastic modulus of 1,880,000 which, as may be observed from the other moduli listed in the foregoing paragraphs, appears to be a characteristic of concrete afflicted with expansive reaction between aggregate and high-alkali cement. The change or decline in elastic modulus has been used by many investigators as a measure of durability, or of the effect produced by some treatment of the concrete and may be considered as reflecting some deterioration. A marked reduction in modulus of elasticity, therefore, is interpreted as a sign of disintegration. Such changes in elasticity will also affect the distribution of stresses in a structure and will result in deflections or strains differing from those assumed in the design.

Measurements of the crown deflections of Copper Basin Dam, made regularly since completion of the structure, by the Metropolitan Water District of Southern California, show a progressive and steady upstream movement of 0.5 in. per year, amounting to 3.75 in. of total displacement in the latter part of 1944 (see Fig. 9). Superimposed upon the record are seasonal fluctuations due to temperature variations and load changes resulting from filling and drawdown of the reservoir; however, the data are positive in regard to a continuous movement of the structure. These deflection measurements have been corroborated by several independent observations from undisturbed control points on the solid-rock abutments and by plumb lines on the downstream face of the dam. Accurate measurements of several chord lengths across the top of the dam likewise reveal enlargement of the concrete, amounting to

0.018% linearly during the period of one full year. Other substantiating evidence of growth in the concrete exists in precise level measurements over points on the top of the dam, which show the crest of the dam to be rising over a 2-yr period. The increase in elevation at various points is roughly proportional to the height of the dam and with reference to this distance indicates a rate of expansion of 0.018% per year, corresponding to the data gathered from the chord measurements. Although these movements are indicative of the general enlargement of the concrete mass, the upstream deflection, which is already large for a thin arch structure, is significant because

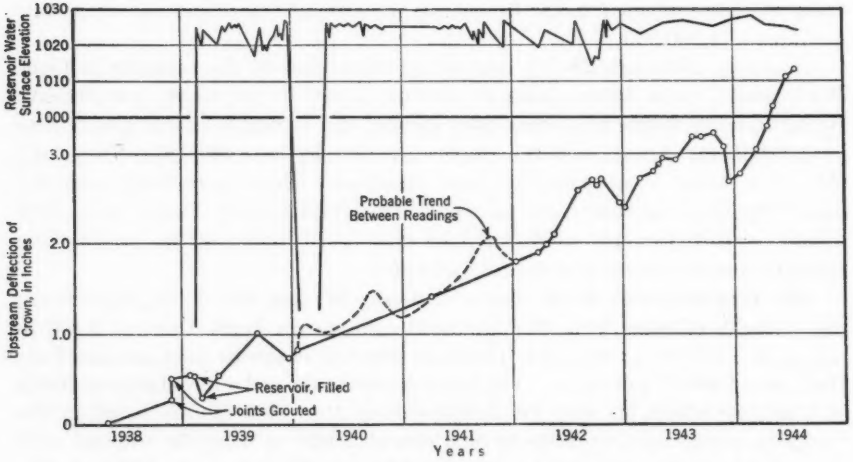

FIG. 9.—THE CROWN OF COPPER BASIN DAM HAS SHOWN PROGRESSIVE UPSTREAM MOVEMENT

of the effect that it may have in changing the stress distribution from that originally contemplated.

Positive evidence of expansion in the concrete of Parker Dam exists in the record of measurements obtained from Invar rods mounted in holes drilled in the concrete. The rods are anchored in the bottom of four adjacent vertical holes (12, 30, 60, and 120 in. deep) and have dial gages attached to their upper ends so that any expansion or contraction over the span of the instruments is registered. As may be seen in Fig. 10, a general progressive expansion is recorded with seasonal temperature variation impressed upon the data from each hole. It is further indicated that expansion of Parker Dam concrete is greater within the interior than near the surface, the shallow holes showing the least expansion.

Aside from surface cracking and general deterioration, reactive expansion in concrete has been most significantly felt through its effect on embedded machinery. The alinement of generator and turbine shafts has been disturbed to the extent that further adjustments required extensive resetting and repairs. The strain in speed rings has been great enough to break the guide vanes, necessitating costly shutdowns and difficult repairs.

Corrective and Preventive Measures

In mild climates expansive cracking may not present the serious aspect that it does in northern regions where it will be aggravated by exposure to the elements. In the latter cases, surface cracks, although superficial, serve as points of entrance for precipitation and assist the destructive processes of freezing and thawing, necessitating repairs much earlier than concrete in more moderate temperatures would require. Such repairs generally would comprise chipping away the affected concrete and incasing the new, taking pains to secure good bond and to provide the new work with joints which would allow additional growth in the underlying mass without producing random cracks in the new work. The paper by Messrs. Sporn and Kammer (1) describes restorations of this nature.

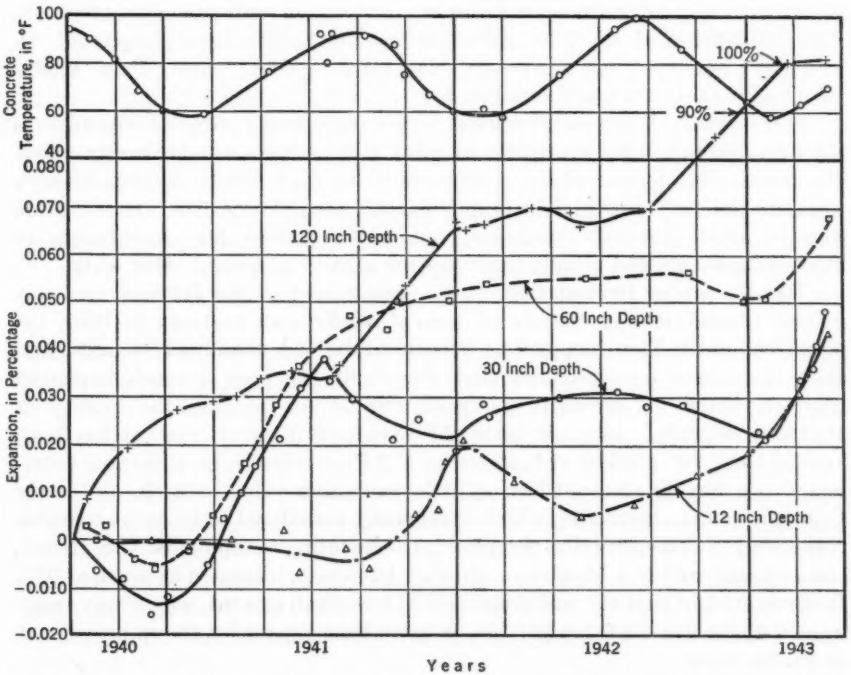

Fig. 10.—Invar Rod Meters, Mounted in Drill Holes, Indicate Progressive Expansion in the Concrete of Parker Dam

Where expansion has affected the alinement of embedded machinery to the extent that its operation can no longer be continued, satisfactory repairs can be effected only by removing the defective concrete in immediate contact with the parts and resetting the machinery in new, sound concrete. Precaution requires that the new concrete be free of the old, with joints or articulations that will insure no disturbance by further movement in the old concrete. Where watertightness in the entire structure must be preserved, the joints

should be provided with flexible metal seals or plastic filler, or both. Continued frequent maintenance, alteration, and adjustment of the embedded machinery itself is the only alternate to any such major restoration work.

New concrete construction in areas where expansive reaction may occur dictates that precautionary measures be taken. The quantities of aggregates required in dam building usually necessitate the utilization of near-by sources for economic reasons. Such sources may have established service records from previous use but more than likely they have not. Therefore, it is usually not feasible to avoid the use of aggregates that may be known to be, or are suspected of being, reactive in combination with high-alkali cements. Furthermore, in most cases, reactive aggregates have been found to possess excellent physical and structural qualities, making them first-class materials for use in concrete in all respects, except in their tendency toward adverse chemical activity in the presence of inordinately high-alkali cements. Experience to date indicates that reactive aggregates can be used without danger of destructive reaction, provided that low-alkali cement (less than 0.60% $Na_2O + K_2O$) is employed (see Fig. 7).

Specification of low-alkali cement for all important hydraulic structures is strongly recommended regardless of what may or may not be known about the reactive tendencies of the aggregates to be used. Such a precautionary measure not only provides insurance against possible eventual deterioration due to alkali-aggregate reactions, but limits undesirable constituents of Portland cements and usually improves the quality in several other ways.

The Bureau of Reclamation, U. S. Department of the Interior, has purchased several million barrels of cement under specifications limiting the alkali content. This cement has been produced by a number of mills, and there is unanimous agreement that the result has been a much improved product, based on presently accepted criteria for judging the quality of Portland cements. In most cases the reduction in alkali content has been accompanied by marked reductions in C_4AF (tetracalcium alumino-ferrite), which is a fluxing material having little cementing value in itself, and lower C_3A (tricalcium aluminate), which is generally considered to be an undesirable compound. Coincidentally the principal cementing compounds, C_3S (tricalcium silicate) and C_2S (dicalcium silicate), have been increased as much as 9%. It seems evident that the additional cost of low-alkali cement, which may range from nothing to about 20¢ per bbl, is more than repaid by the improvement in quality alone.

Tests have indicated that some Portland pozzolanic cements may be safely used with reactive aggregate. It has also been demonstrated that additions of fine pozzolanic materials to reactive aggregate and high-alkali cement combinations or a replacement of some of the high-alkali cement by the pozzolanic material will reduce the expansion. In the construction of Friant Dam, on the San Joaquin River in California, 20% of naturally fine pumicite (by weight of the cement) was used as one of the concrete ingredients. Both high-alkali and low-alkali cements were used in the concrete since wartime production conditions made it impossible to secure low-alkali cement for all the work. The aggregate used in this dam was found to be reactive with high-alkali

cement and it is therefore felt that pumicite has furnished some protection against expansion where the low-alkali cement was not provided.

No specific preventive for alkali-aggregate reaction has as yet been discovered although the problem is under intensive investigation. Specification of low-alkali cement, while admittedly inadequate, is the only recourse available to engineers at the present time.

Summary

Expansive reaction between aggregate and high-alkali cement has been found responsible for the random cracking and deterioration of the concrete of a large number of dams. The binding of gates and of generator rotors,the misalinement of generator and turbine shafts, and the straining and breakage of guide vanes and other machinery embedded in concrete have been traced to the excessive expansion which accompanies such reaction. Aggregates that have been found to react with high-alkali cement include opaline silica, highly siliceous rocks (such as siliceous limestones, chalcedony, and some cherts), and acid to intermediate volcanic rocks. These same materials have caused no trouble in service when combined with cement low in alkali content.

Aggregate which will give this trouble in concrete may be recognized if it produces expansion within a short time in test specimens made with high-alkali cement. Absence of such expansion in a short period is no guarantee, however, against a latent type of reaction, for which no reliable accelerated test has appeared. Prudence dictates that aggregates known or suspected of adverse activity be used only with cement of low-alkali content or not at all. Low-alkali cement is recommended generally for all concrete in hydraulic structures as its use provides security against the uncertainty of encountering expansive reaction, and otherwise improves the general quality of the concrete.

Acknowledgments

Acknowledgment is due Julian Hinds, M. Am. Soc. C. E., general manager and chief engineer, Metropolitan Water district of Southern California, for the data on deflection measurements at Copper Basin Dam; and Raymond E. Davis, M. Am. Soc. C. E., director, Engineering Materials Laboratory, University of California, for encouragement and advice. Activities of the Bureau of Reclamation are supervised by Harry W. Bashore, M. Am. Soc. C. E., commissioner; and all engineering and construction are under the direction of S. O. Harper, M. Am. Soc. C. E., chief engineer. Research and design work are directed by J. L. Savage, Hon. M. Am. Soc. C. E., chief designing engineer.

APPENDIX

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PAPERS

THE EFFECT OF BLANKETS ON SEEPAGE THROUGH PERVIOUS FOUNDATIONS

BY PRESTON T. BENNETT,[1] ESQ.

SYNOPSIS

A mathematical analysis of seepage through natural or artificial blankets of relatively impervious soil overlying a pervious foundation is presented in this paper, indicating methods for the application of formulas to the design of earth dams. Since natural foundations rarely if ever approach the simplicity of structure and the uniformity assumed in this analysis, the blanket formulas should not be considered as an accurate means of computing flow conditions, but as an aid to the judgment of the designer.

INTRODUCTION

In the design of earth dams built on pervious foundations, there appears to be a trend toward the control of seepage through the foundation by construction of an upstream blanket connected to the impervious section of the dam, and providing drainage for the downstream section of the dam by horizontal pipe drains or by a system of vertical drainage wells. In dams where the control of seepage is undertaken primarily to increase the stability of the structure, rather than to reduce seepage losses to a minimum, this design offers several advantages over a central impervious section in the dam, and a cutoff through the foundation.

In the latter design, practically all the hydrostatic pressure of the reservoir is exerted on the central core and cutoff, and the downstream half of the dam must have slopes sufficiently flat to withstand the full pressure of the reservoir. If the reservoir head is dissipated through a blanket and upstream impervious section, a much larger portion of the dam is effective in offering resistance to the hydrostatic pressure. Furthermore, the flow of water through the upstream impervious section is almost vertically downward toward the pervious foundation, which is maintained at a relatively low hydrostatic potential by the combined action of the upstream blanket and the downstream drainage system.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by June 1, 1945.

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There is little tendency, upon sudden drawdown of the reservoir, for the water saturating the upstream section to flow back into the lowered reservoir, and hence the adverse conditions resulting from drawdown are reduced to a minimum. If an excess of impervious material unsuitable for embankment construction is available, the most important advantage of the blanket lies in the economic use of waste material, rather than in the use of a costly cutoff wall.

Quite commonly the alluvial deposit in a river valley consists of a stratum of sand and gravel overlain by a surface layer of relatively impervious finer soils. It is advantageous to leave such a blanket in place for seepage control, but it may be desirable, on the other hand, to remove a part of it to reduce the haul from upstream borrow pits. If the natural blanket is sufficiently uniform in thickness and permeability to justify the application of blanket design theory, the formulas developed herein may be of assistance to the designer in establishing limits for borrow-pit excavation, which will not reduce the blanketing effect materially.

The mathematical analysis of flow through blankets presented herein is of necessity based on idealized assumptions as to the uniformity of permeability and thickness of the pervious stratum and blanket. As in any other phase of soil mechanics, the validity of any conclusions based on this analysis is limited by the degree to which these idealized conditions are approached in nature, and likewise by the degree of accuracy obtainable in the determination of the permeability of the soils involved.

Since natural deposits of soils cannot be expected to be uniform in thickness or permeability, and since the determination of the permeability of such deposits can result at best only in a good approximation, the formulas in the paper are not intended to represent, accurately, the actual conditions of flow and potential distribution under a dam. Nevertheless, if several values for the permeability and thickness of the blanket and pervious stratum are used, covering the estimated range of variation of these factors, the resulting analytical evaluation of a blanket should be of value as an indication of the probable limits of its effectiveness.

Notation

The letter symbols in this paper are defined where they first appear, in the text or by illustration, and are assembled for convenience of reference in Appendix I.

Assumptions

The following derivation is applicable only when the ratio of foundation permeability to blanket permeability is large—say 10 to 1 or greater. If the contrast in permeability is small, the assumption that flow is vertical through the blanket and horizontal through the foundation may not be warranted. Furthermore, it has been assumed that the vertical plane through the upstream end of the blanket is an equipotential surface at full reservoir head. Actually, the equipotential plane coincides with the bottom of the reservoir, and in the cases represented subsequently by Eqs. 9*a*, 12*a*, and 18*a*, a small loss of head occurs between the true and assumed equipotential surfaces. However, if the

length of the path of flow through the foundation is great as compared to the depth of the foundation, this head loss is negligible as compared to the errors due to nonuniformity of natural soils. The assumptions relative to direction of flow and potential distribution at the boundaries are shown in Fig. 1.

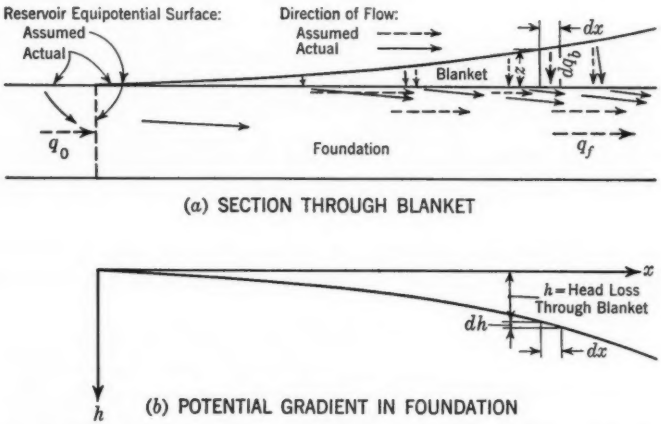


FIG. 1.—ASSUMED DIRECTION OF FLOW, AND POTENTIAL DISTRIBUTION AT THE BOUNDARIES

GENERAL CASE

At any point x, under the blanket (Fig. 2), the horizontal flow through the foundation is equal to the flow through the blanket upstream from that point, plus the inflow under the upstream end of the blanket. That is:

$$q_f = \int_0^x dq_b + q_{f0} \quad \ldots\ldots (1)$$

The vertical flow dq_b, through a small element of the blanket, of width dx and thickness z_b, varies directly with the head loss h, and inversely with blanket thickness z_b, so that

$$dq_b = \frac{k_b\, h\, dx}{z_b} \quad \ldots\ldots (2)$$

In the interval dx, the horizontal flow in the pervious stratum is increased by an amount equal to the vertical inflow dq_b, so that

$$\frac{dq_f}{dx} = \frac{dq_b}{dx} = \frac{k_b\, h}{z_b} \quad \ldots\ldots (3)$$

Since

$$q_f = k_f\, z_f \frac{dh}{dx} \quad \ldots\ldots (4a)$$

it follows that

$$\frac{dq_f}{dx} = k_f\, z_f \frac{d^2h}{dx^2} \quad \ldots\ldots (4b)$$

and, from Eq. 3,

$$\frac{d^2h}{dx^2} = \frac{k_z\, h}{z\, k_f\, z_f} \quad \ldots\ldots (4c)$$

Using the notation $a = \sqrt{\frac{k_z}{z_b\, k_f\, z_f}}$ for blankets of uniform thickness, and $\alpha = \frac{k_z}{k_f\, z_f}$ for blankets of variable thickness, Eq. 4c may be written:

$$\frac{d^2h}{dx^2} = a^2\, h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5a)$$

or

$$\frac{d^2h}{dx^2} = \frac{\alpha\, h}{z_b} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5b)$$

For a blanket of constant thickness and infinite horizontal extent, the solution to Eq. 5a is

$$h = h_0\, e^{ax} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which h_0 is the head loss through the blanket at point $x = 0$, which is taken for convenience in this case at the downstream end of the blanket. This

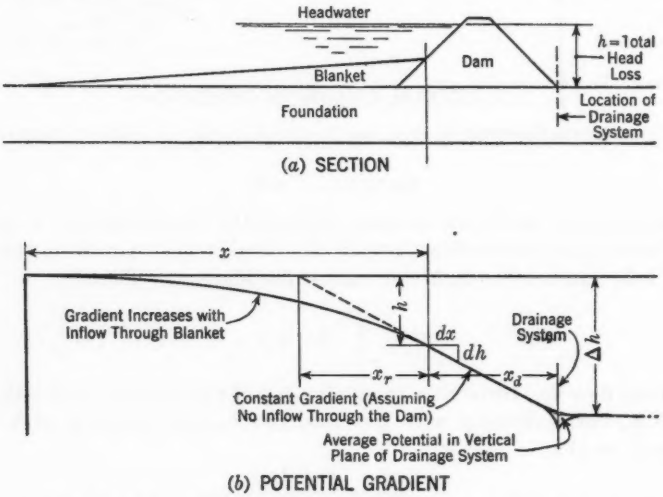


FIG. 2.—RELATION BETWEEN RESISTANCE, HEAD LOSS, AND GRADIENT

solution is verified by successive differentiation:

$$\frac{dh}{dx} = a\, h_0\, e^{ax} = a\, h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7a)$$

and

$$\frac{d^2h}{dx^2} = a^2\, h_0\, e^{ax} = a^2\, h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7b)$$

The resistance x_r is a measure of the efficiency of a blanket. It may be defined as the length of a prism of the foundation material, of thickness z_f and permeability k_f, which, under head loss h, would carry a flow equivalent to the flow which passes the blanket system under the same head loss. The relation between resistance, head loss, and gradient is shown graphically in Fig. 2.

Defined algebraically, $x_r = \dfrac{h}{\dfrac{dh}{dx}}$; or, for this case,

$$x_r = \frac{h}{a\,h} = \frac{1}{a} = \sqrt{\frac{z_b\,k_f\,z_f}{k_z}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Eq. 8 may be used to estimate the effectiveness of a natural surface layer as a blanket when none of the fine material is removed for borrow, and all the seepage enters the substratum by percolation through the blanket.

If part of a natural blanket is removed, the substratum is exposed directly to full reservoir head, and a part of the flow enters the foundation sand at the upper limit of the blanket. For this case, the solution of Eq. 5*a* is

$$h = h_n\,(e^{ax} - e^{-ax}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9a)$$

$$\frac{dh}{dx} = a\,h_n\,(e^{ax} + e^{-ax}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9b)$$

$$\frac{d^2h}{dx^2} = a^2\,h_n\,(e^{ax} - e^{-ax}) = a^2\,h \quad \ldots\ldots\ldots\ldots\ldots\ldots (9c)$$

and

$$x_r = \frac{e^{ax} - e^{-ax}}{a\,(e^{ax} + e^{-ax})} = \frac{e^{2ax} - 1}{a\,(e^{2ax} + 1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9d)$$

In Eqs. 9, $x = 0$ at the upstream edge of the blanket, and h_n is a constant depending on the total head loss of the system of which the blanket is a part, and on the ratio of the blanket resistance to the remainder of the system. The value of h_0 in Eq. 7*a* is determined in the same manner. For example, assume a top stratum varying in thickness from 10 ft to 15 ft, overlying a sand foundation 50 ft to 60 ft thick, and assume that the ratio of blanket permeability to foundation permeability varies between 0.005 and 0.01. The extreme values of a, as computed from these dimensions, are 0.00246 and 0.0045, corresponding to resistances of 425 ft and 225 ft. If Δh and x_d (see Fig. 2) are, respectively, 80 ft and 750 ft for the dam in question, and, if 300 ft is adopted as the average resistance of the blanket, the effective total length of the percolation path is 1,050 ft, and the head loss through the blanket is $300/1{,}050 \times 80$, or 23 ft. Using $a = 0.0033$, corresponding to $x_r = 300$, and $h_0 = 23$, Eq. 7*a* becomes, in this case,

$$h = 23\,e^{0.0033x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The value of h_n, in Eq. 9*a*, is determined after computation of a definite value of h at the lower end of the blanket, using the relation between Δh, x_d, and x_r indicated graphically in Fig. 2 and as illustrated in the preceding example.

Ordinarily, for upstream blankets, the pressure distribution under the blanket is not of great interest, and the computation of constants for Eq. 6 or 9*a* need not be made, since the determination of x_r is all that is necessary for the estimation of pressures under the dam, or for computation of the approximate horizontal flow or discharge per linear foot of dam, which is

$$q_f = \frac{k_f\,z_f\,\Delta h}{x_r + x_d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Under downstream blankets, however, the substratum pressures are of considerable importance, and Eqs. 6 and 9*a* may be used in the design of the downstream drainage system to estimate the pressures that would exist if no drainage were provided, and if the underseepage were forced to find its way to the surface against the resistance of an impervious top stratum. When used for downstream blankets, the head loss h is referred to the ground surface (assumed level) downstream from the dam, or to tailwater elevation if the downstream toe is submerged.

The curves in Fig. 3 give values of x_r computed from x and a by the use of Eq. 9*d*. The curves for x_r and $\frac{z_b\, x}{\alpha}$ are plotted on the a-scale and the x-scale; and the curves for z_b are plotted on the a-scale and the α-scale. Suppose, in the

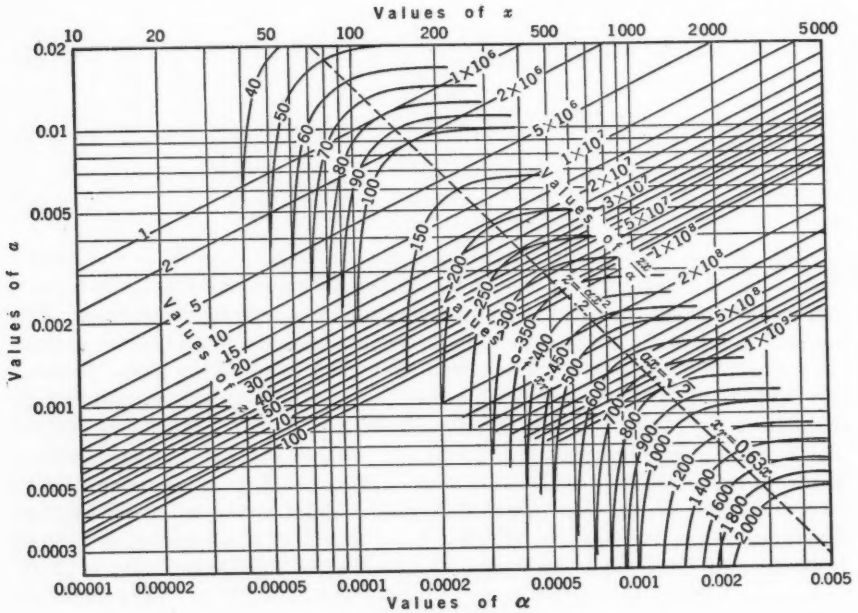


FIG. 3.—RESISTANCE CURVES

foregoing example, that it is desired to excavate the natural blanket for use as fill material, leaving in place enough to provide a resistance of 200 ft. Using the higher of the limiting values of a (Fig. 3), the curve $x_r = 200$ intersects $a = 0.0045$ at $x = 330$ ft, the length of blanket to be left in place.

It is apparent from the curves of Fig. 3 that the ratio of resistance to blanket length, for any value of a, decreases rapidly as the length is extended beyond that indicated by the diagonal line $a\,x = \sqrt{2}$ in Fig. 3, and that very little benefit is to be gained either by leaving a natural blanket in place to great distances above a dam, or by the construction of an artificial blanket of uniform thickness and excessive length. These considerations lead to the conclusion

that, if a blanket is to be built, its thickness probably should be reduced at the upstream where the blanket is relatively ineffective, and increased near the dam, interposing an additional thickness of impervious material where the head loss through the blanket is greatest.

This procedure would result in a blanket of triangular section, for which the resistance may be determined by substituting $s\,x$ for z_b in Eq. 5*b*, s being the slope of the blanket surface, and x the blanket length measured from its upstream edge. The solution for this case, as developed by Stanley U. Benscoter, Jun. Am. Soc. C. E., is as follows:

$$h = h_n \left\{ x + \frac{\alpha\, x^2}{2\, s} + \frac{\alpha^2\, x^3}{2!\, 3!\, s^2} + \cdots + \frac{\alpha^{(n-1)}\, x^n}{(n-1)!\, n!\, s^{(n-1)}} \right\} \ldots\ldots (12a)$$

$$\frac{dh}{dx} = h_n \left\{ 1 + \frac{\alpha\, x}{s} + \frac{\alpha^2\, x^2}{[2!]^2\, s^2} + \cdots + \frac{\alpha^{(n-1)}\, x^{(n-1)}}{[(n-1)!]^2\, s^{(n-1)}} \right\} \ldots\ldots (12b)$$

and

$$\frac{d^2h}{dx^2} = h_n \left\{ \frac{\alpha}{s} + \frac{\alpha^2\, x}{2\, s^2} + \cdots + \frac{\alpha^{(n-1)}\, x^{(n-2)}}{(n-2)!\, (n-1)!\, s^{(n-1)}} \right\} \ldots\ldots (12c)$$

If Eq. 12*c* is multiplied by $\frac{s\,x}{\alpha}\left(=\frac{z_b}{\alpha}\right)$, the right-hand side becomes identical with the value of h in Eq. 12*a*, and the resulting equation

$$\frac{d^2h}{dx^2}\,\frac{z_b}{\alpha} = h \ldots\ldots (13)$$

verifies Eq. 12*a* as a solution of Eq. 5*b* for the triangular blanket.

The value of x_r, obtained by dividing Eq. 12*a* by Eq. 12*b*, is

$$x_r = x - \frac{\alpha\, x^2}{2\, s} + \frac{\alpha^2\, x^3}{3\, s^2} - \frac{11\, \alpha^3\, x^4}{48\, s^3} + \frac{19\, \alpha^4\, x^5}{120\, s^4} \ldots\ldots (14)$$

This series converges very slowly, and is not adapted to numerical computations. Eqs. 12*a* and 12*b* are somewhat better suited to numerical work, and x_r can be obtained from h and $\frac{dh}{dx}$ as computed from these series. Using this method, values of x_r for several triangular sections have been computed, and compared with the resistances of uniform sections of equal end area. From these comparisons, it appears that the resistance of a triangular section is approximately equal to that of a rectangular section of the same end area, if the length of the latter is one half the length of the triangular section.

The numerical difficulties involved in computing the resistance of a triangular section can be avoided by substituting a vertically curved surface profile,

$$z_b = \frac{\alpha\, x^2}{m\,(m-1)} \ldots\ldots (15)$$

for the tangent profile $z_b = s\,x$. This substitution is made only to facilitate computations, and it is not recommended that blankets be constructed to fit the curves used for purposes of analysis. For the curved surface indicated by

Eq. 15 the solution to the general formula (Eq. 5*b*) is

$$h = h_n x^n \quad \ldots\ldots (16a)$$

$$\frac{dh}{dx} = n\, h_n\, x^{(n-1)} \quad \ldots\ldots (16b)$$

and

$$\frac{d^2h}{dx^2} = n\,(n-1)\, h_n\, x^{(n-2)} \quad \ldots\ldots (16c)$$

The solution is verified by substituting values of z_b and h from Eqs. 15 and 16*a* in Eq. 16*c*. From Eqs. 16*a* and 16*b* the resistance is

$$x_r = \frac{x}{n} \quad \ldots\ldots (17)$$

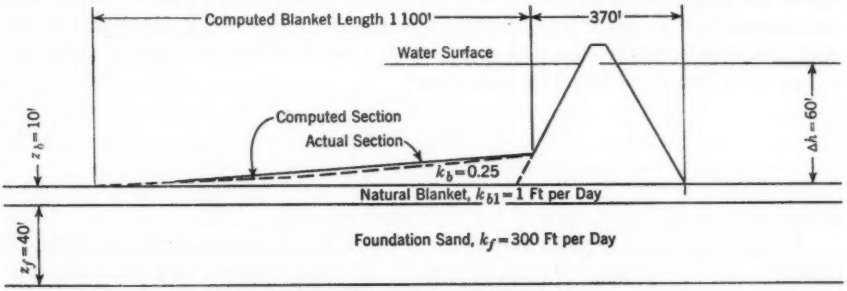
The use of Eqs. 15 and 17 to approximate the resistance of a triangular section is shown by the following examples, computed for a blanket 500 ft long, which varies uniformly from 0 to 10 ft in thickness, and is built on a pervious stratum 40 ft thick, of such permeability that $\frac{k_b}{k_f} = 0.005$. For these conditions, $\alpha = \frac{k_b}{k_f z_f} = 0.000125$ and a curved section conforming to Eq. 15 will have a thickness of 10 ft at $x = 500$ ft when $n\,(n-1) = 3.12$, or $n = 2.34$. The resistance of this section is 500/2.34 = 214 ft. Another approximation of the triangular section may be made by selecting n (Eq. 15) so that the end area of the parabolic section equals that of the triangular shape. For the preceding example, this gives $n = 2.04$ and $x_r = 245$ ft. Using the assumption that the triangular section 500 ft by 10 ft is equivalent to a uniform section 250 ft by 10 ft, Fig. 3 gives $a = 0.0035$ (for $z_b = 10$ and $\alpha = 0.000125$) and $x_r = 200$ when $x = 250$.

Considering the fact that the basic data used in the computations will be only approximately correct, the foregoing approximate values of the resistance of a triangular section are in sufficiently close agreement to justify the use of either Eq. 9*d* or 17 in estimating the efficiency of this type of blanket. The approximate relationship between rectangular and triangular sections, thus far developed, however, does not furnish any answer to the question raised as to the relative merits of the two sections. Referring again to Fig. 3, it will be seen that the diagonal lines that represent constant blanket thickness, when referred to the α-scale and the a-scale, also represent constant values of the sectional area $z_b\, x$ when referred to the x-scale and the a-scale on which the x_r-curves are plotted. These constant-area curves may be used to estimate the optimum dimensions of a rectangular section if the quantity of blanket material is fixed. For example, suppose that 20,000 cu yd of blanket material per station is available, and that its permeability is such that $\alpha = 0.0001$. The sectional area is then 5,400 sq ft, and $\frac{z_b\, x}{\alpha} = 5.4 \times 10^7$. The maximum resistance (300 ft) is found at $x = 475$, and the corresponding thickness is 11.4 ft. If the blanket material were much more pervious, so that $\alpha = 0.0027$ and $\frac{z_b\, x}{\alpha} = 2 \times 10^6$, the

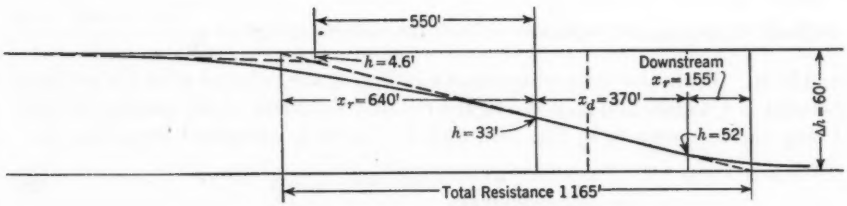
optimum length would be 160 ft, the thickness 34 ft, with a resistance of 100 ft. However, by following the line $\frac{z_b x}{\alpha} = 2 \times 10^6$ to intersect $x_r = 90$ at $x = 105$ and at $x = 250$, it is seen that the optimum length may be reduced by about 34% or increased by about 56% with a loss of only 10% in resistance, if the thickness is correspondingly changed to retain the same sectional area. Since the x_r-curves are spaced to conform to the logarithmic x-scale, the same ratio of loss of resistance to increase or decrease from the optimum value of x holds true for any value of x on the broken line $a\,x = \sqrt{2}$.

Since Fig. 3 is approximately applicable to triangular sections if the base length is considered to be $2\,x$, it appears that either the triangular or rectangular shape may be used with equal effectiveness, and that for either section a considerable departure may be made from the theoretical optimum dimensions without great loss of efficiency. It is concluded, therefore, that feasibility of construction should govern the dimensions of a blanket, as long as the extremes of inefficiency represented by the ends of the x_r-curves in Fig. 3 are avoided.

For a combination of natural and artificial blankets, such as is shown in Fig. 4(*a*), it is again convenient to represent the triangular section by an



(*a*) SECTION THROUGH DAM



(*b*) COMPUTATION OF HEAD IN SUBSTRATUM

FIG. 4.—ILLUSTRATIVE EXAMPLE WITH IMPERVIOUS DAM

approximately equivalent parabolic section to simplify the computations. It can be shown analytically that, if $n = 2$ in Eq. 15, the maximum resistance can be obtained from a fixed sectional area. (It is of interest to note that the relationship, between z_b and x for the most favorable ratio of $\frac{x_r}{z_b\,x}$, is identical

for the curved section as obtained analytically, and for the rectangular section as obtained graphically by the curves in Fig. 3.) The arbitrary use of $n = 2$ simplifies computation, and at the same time permits the development of a formula adapted to use in designing combination blankets. Let the vertical gradient $\frac{h}{z_b}$ be a constant, i. Then

$$z_b = \frac{h}{i} = \frac{\alpha x^2}{2} + \beta x + z_{b0} \quad \ldots\ldots (18a)$$

$$\frac{dh}{dx} = i(\alpha x + \beta) \quad \ldots\ldots (18b)$$

$$\frac{d^2h}{dx^2} = i\alpha = \frac{\alpha h}{z_b} \quad \ldots\ldots (18c)$$

and

$$x_r = \frac{z_b}{\alpha x + \beta} \quad \ldots\ldots (18d)$$

At the upstream end of this section $x = 0$, $h = i z_{b0}$, and $\frac{dh}{dx} = i\beta$. If the upstream end adjoins the lower end of another blanket for which the head loss and foundation gradient are numerically equal to $i z_{b0}$ and $i\beta$, there will be no discontinuity in the foundation flow. This condition may be met by choosing values of β and z_{b0} in Eq. 18*a* such that

$$\frac{z_{b0}}{\beta} = \frac{h}{\frac{dh}{dx}} = x'_r \quad \ldots\ldots (19)$$

in which x'_r is the resistance of the upstream blanket.

An application of Eqs. 18 and 19 is shown in Fig. 4, representing an artificial blanket, whose permeability k_{b1} is 0.25 ft per day, constructed on a natural blanket 10 ft thick with a permeability $k_{b2} = 5$ ft per day. The natural blanket may be replaced, for purposes of computation, by one having a thickness of 0.5 ft and a permeability of 0.25 ft per day, equal to that of the artificial section, without changing its resistance, which for infinite length is $\sqrt{\frac{10 \times 300 \times 40}{5}}$ or 155 ft. From the dimensions shown in Fig. 4, the value of α for the artificial blanket is 1/48,000 and z_{b0} is 0.5 ft, the reduced thickness of the natural blanket. Using these constants in Eq. 18*a*, and $\beta = 0.0032$, computed from Eq. 19,

$$z_b = \frac{x^2}{96{,}000} + 0.0032\,x + 0.5 \quad \ldots\ldots (20)$$

in which z_b includes the reduced thickness of the blanket already in place. If the size of the blanket is limited by availability of material to a sectional area of 6,000 sq ft for the case under consideration, the length is found to be about 1,100 ft from Eq. 20, and the corresponding thickness at the dam is 16.2 ft, or 16.7 ft, including the reduced thickness of the original surface layer. From Eq. 18*d*, the resistance is $\frac{16.7}{0.023 + 0.003}$ or 640 ft, which includes the effect of the natu-

ral blanket upstream from the point where no additional material is placed. From Fig. 4(*a*), the length of the path of flow under the dam is 370 ft, and the resistance of the system 1,165 ft if it is assumed that no seepage takes place through the dam proper. The pressure gradient in the foundation, as determined by these computed resistances, is shown in Fig. 4(*b*). In this example it has been assumed for the sake of simplicity that the entire dam is impervious, and that all the seepage must escape through the downstream blanket. In actual design, foundation drainage would ordinarily be provided in the form of a pervious drainage blanket within the dam, a system of pressure relief wells, or a horizontal pipe drain, and the path of flow or resistance downstream from the blanket would be less than in the example given. A discussion of downstream drainage facilities is outside the scope of this paper, but, since a knowledge of the downstream resistance is necessary in evaluating the effectiveness of a blanket, approximate methods for estimating the path of flow under the dam are included in Appendix II.

As previously stated, the curved section, shown by the dotted line in Fig. 4(*a*), was adopted for purposes of computation only, and would be replaced in actual construction by an equivalent section of similar shape, as indicated. Theoretically, the vertical gradient is constant over the entire length of the curved section, and therefore the curve represents an efficient and well-balanced section which should serve as a good guide in the selection of the actual blanket shape adopted for construction.

Subject to the limitations previously stated as to accuracy of the basic data used in the computations, the formulas for uniformly thick blankets may be used to estimate the substratum pressures downstream from a dam when no drainage is provided, as in the section shown in Fig. 4(*a*). In this example, although the natural blanket has a relatively low resistance, the computations indicate the desirability of a downstream drainage system, and at the same time provide a method whereby the discharge from the system may be estimated.

Summary

The foregoing analysis of flow through blankets is based on assumed conditions which are approached very closely when the ratio of foundation to blanket permeability is large, and when the length of the horizontal path of flow is large as compared to the thickness of the pervious foundation. When these conditions are met, the formulas are mathematically exact for practical purposes.

The difficulty of accurate determination of the permeability of the soils in place, and the nonuniformity of thickness and permeability of natural deposits, constitute the greatest source of error in use of this analysis in design. However, since the same sources of error are present in analysis of flow by models, the analytical method is to be preferred for blanket studies when the foundation may be assumed to consist of a single pervious layer. If the foundation is separated into two or more distinct strata by impervious lenses, a model study would be the better method to investigate the resulting complex flow pattern.

In view of the uncertainties as to the thickness and permeability of natural strata involved, it might first appear that the methods of analysis presented

herein are unnecessarily elaborate. The methods have been developed in considerable detail, however, in the belief that the general principles involved, in the design of blankets, could best be presented by a mathematical analysis, and that the application of these principles to blanket problems, although not leading to exact results, will provide, in many cases, an improved method of estimating blanket performance.

APPENDIX I

NOTATION

The following letter symbols, used in this paper, conform essentially with "Soil Mechanics Nomenclature"[2] adopted by the Society's Board of Direction in 1941:

a = an abbreviated notation for $\sqrt{\frac{k_b}{z_b\, k_f\, z_f}}$;

α = an abbreviated notation for $\frac{k_b}{k_f\, z_f}$;

β = a constant, equal to the slope of a curved blanket at $x = 0$;

e = base of natural logarithms;

h = head or potential at any point, x, in the foundation (referred to reservoir potential) and equal to the head loss through the blanket at point x; h_1, h_2, etc. = head at specified points in the foundation:

$\frac{dh}{dx}$ = horizontal potential gradient in the foundation;

$\frac{d^2h}{dx^2}$ = rate of change of $\frac{dh}{dx}$, proportional to inflow through the blanket;

h_n = a constant factor for computing h, determined by the relationship between the blanket and the flow system as a whole;

h_0 = head loss through the blanket at point $x = 0$;

i = vertical potential gradient through blanket, for special case of constant gradient;

m = an arbitrary constant defining the profile of curved blanket surfaces;

q = flow per unit width; q_1, q_2, q_3, etc. = flow through designated sections of the blanket or foundation;

q_b = vertical flow per unit width, through the blanket; d_{qb} = element of vertical flow through an element dx of the blanket;

q_f = horizontal flow, per unit width, through the foundation;

q_0 = inflow to the foundation under the upstream end of the blanket;

$s = z/x$, the slope of the surface of a triangular blanket;

x = horizontal distance measured positive in the direction of flow; dx = an element of the distance x;

x_r = resistance of a blanket, being the effective length of the path of flow under the blanket;

[2] *Manual of Engineering Practice No. 22*, Am. Soc. C. E., 1941.

z = thickness of a blanket, when the thickness is variable;
z_0 = thickness of blanket at $x = 0$;
z_b = thickness of a blanket when the thickness is constant; and
z_f = thickness of the foundation stratum.

APPENDIX II

Flow Under the Dam Proper

A principle which may be used, in some cases, to estimate the characteristics of flow under the dam itself is illustrated by analogy with the flow under the composite blanket shown in Fig. 4(*a*). Controlling points and slopes on the pressure gradient curve of Fig. 4(*b*) may be obtained without recourse to the equations defining h in terms of x, as follows: Assign any convenient value, say, $h_1 = 1.55$ ft to the head loss through the natural blanket at its junction with the artificial blanket. Under this head, the foundation gradient at this point is $1.55/155 = 0.01$, since the natural blanket resistance is 155 ft, and $q_1 = 0.01\ k_f z = 120$ cu ft per day. For this particular case, the vertical gradient through the

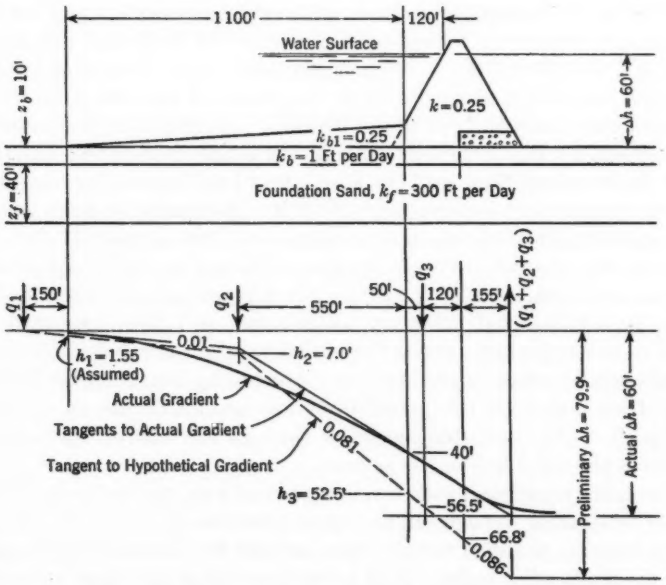


Fig. 5.—Illustrative Example with Flow Through Dam

artificial blanket is constant throughout its length, and is equal to the maximum gradient through the natural blanket. In terms of the reduced thickness of 0.5 ft, the constant vertical gradient is $1.55/0.5 = 3.1$, and the flow through the artificial blanket is $q_2 = 3.1 \times 0.25 \times 1{,}100 = 853$ cu ft per day, making the

total foundation flow at the lower end of the blanket system 973 cu ft per day, with a corresponding horizontal gradient of $\frac{q_1 + q_2}{k_f z_f} = 0.081$. For purposes of computing the head loss at the lower end of the blanket system, the uniform inflow through the artificial blanket may be considered concentrated at its midpoint, and the inflow through the natural blanket as concentrated at a point 155 ft upstream from its lower end. For this assumed condition, the foundation gradient would be two tangents to the actual gradient curve, intersecting at the midpoint of the artificial blanket as shown in Fig. 4(*b*), and the numerical value of $h_2 = 52.5$, corresponding to the assumed value of $h_1 = 1.55$, may be computed from the tangent slopes as indicated in Fig. 5.

In Fig. 5, the blanket is identical with that in Fig. 4, but the dam section consists of an upstream "impervious" section having the same permeability as the blanket, and has a downstream drainage blanket of very high permeability, such that seepage entering it from the foundation will be carried off without appreciable resistance, and the drainage blanket may be assumed to be at tailwater potential. The flow through the impervious upstream section will be principally downward, and the vertical gradient may be roughly estimated as a mean between the value at the lower end of the blanket and the value at the line of saturation, where the vertical gradient for gravity flow may be taken as 1.0. On the basis of these assumptions, the average downward gradient through the dam is 2.0, and over a horizontal distance of 120 ft as shown in Fig. 5, the inflow q_3 is $2 \times 0.25 \times 120 = 60$ cu ft per day. The "center of gravity" of the inflow is somewhat upstream from the center of the 120-ft interval—say 50 ft downstream from the end of the blanket. At this point the hypothetical gradient shown by the broken line in Fig. 5 changes from 0.081 to 0.086 as a result of the increased flow, and the foundation head loss at the upstream end of the drainage blanket is accordingly 66.8 ft. Assuming no back pressure in the drainage blanket of the dam, the resistance of the natural blanket remains 155 ft and in this distance downstream the additional head loss, computed from the gradient of 0.086, is 13.5 ft making a total hypothetical head loss of 79.9 ft resulting from the initial arbitrary assumption of 1.55-ft loss through the upstream natural blanket. Since the actual head on the system is 60 ft, the preliminary head losses and gradient are corrected by the factor 60/79.9 to give the controlling points on the actual head loss profile, as shown by the solid line in Fig. 5. The computed head loss through the blanket system is 39 ft, or two thirds the total loss of the system.

The foregoing example illustrates, in a general way, the methods which may be used for estimating the over-all flow characteristics of a pervious foundation, and which may be modified to take into account toe drainage by longitudinal drains or pressure relief wells. It is to be noted that the effect of horizontal flow in the dam has been neglected entirely in this approximation, and that this method is not applicable where the flow is substantial as compared to the seepage through the foundation.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

UTILIZATION OF GROUND-WATER STORAGE IN STREAM SYSTEM DEVELOPMENT

BY HAROLD CONKLING,[1] M. AM. SOC. C. E.

SYNOPSIS

The scope of this paper is limited to consideration of those formations which may be called underground reservoirs, into which water is charged naturally or can be charged artificially when there is a surplus, and from which water is available when needed either as gravity escape from the ground-water reservoir into a surface stream (rising water or return flow) or as controlled extractions by wells and pumps.

INTRODUCTION

The first developers of irrigation on the stream systems in western United States utilized such surface discharges as were available. In streams fed by melting snow these discharges might be fairly large until late in the growing season, but in streams where snowfall at the headwaters was small, or where the runoff season began early, the dependable surface flow was small in the latter part of the growing season. In this circumstance and where climate and geology were favorable, recourse was had at about the turn of the century to wells and pumps to utilize ground water for supplementary irrigation; but in locations where runoff was larger and in more favorable regimen, utilization of ground water was long delayed unless artesian flows could be developed. However (especially in the 1930's which in western United States was a decade of drought generally), pressure of need forced the use of ground water by individuals even in those areas favored by fairly stable surface flow in the irrigation season and by large surface reservoir capacity. Use of pumped water for irrigation is expanding rapidly in some of these areas and in others that formerly had available only such natural discharge as occurred in the flood season of early summer.

Complete development of water supplies for irrigation in the United States will require the greatest possible use of the underground portion of the streams

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by June 1, 1945.

[1] Deputy State Engr., Water Rights, Sacramento, Calif.

and of the storage capacity resident in the unconsolidated alluviums of the stream valleys. The plans for large development by federal government agencies in cooperation with state and local agencies contemplate this use of underground sources. One of the principal features of the Central Valley Project in California is the use of underground reservoirs (17)(21).[2]

Such utilization is discussed in this paper from an engineering viewpoint and a brief review is presented of the somewhat inchoate body of knowledge of the subject. This paper is a preliminary attempt to correlate the subject matter, with emphasis upon those phases most useful to an engineering approach. Omission of other phases of the hydrology and geology of underground water does not deny their importance for other than the matters discussed herein nor their possible importance even in these matters.

The object of the paper places limitations on its scope:

1. Only that part of the United States west of the 97th to 100th meridian is included. This is the arid or semiarid portion, in practically all of which irrigation is needed for successful intensive agriculture.
2. Only those situations in which the following conditions exist are considered:

(*a*) Ground water in the alluvium is readily accessible to extraction by pumps;
(*b*) The alluvium is sufficiently permeable to yield water in commercial quantities;
(*c*) The alluvium is naturally charged with water or is susceptible of being charged artificially to such an extent that heavy commercial drafts can be sustained; and
(*d*) If artificial charging is necessary it can be done at feasible cost.

The foregoing conditions are necessary for the successful use of ground water in irrigation.

The scope of the paper excludes the lava beds of Idaho, Oregon, and Washington and the limestone formations of New Mexico. Immense quantities of water might be secured from the lavas judging from the large springs that discharge from the formation, and considerable quantities are being secured from limited areas of the limestone; yet experience is small concerning the exploitation of the ground water of the lavas by wells, and such exploitation would be generally hazardous and costly. The limestone formations are limited in extent, and utilization of their waters is still more limited.

The criteria eliminate from consideration almost all of the water-bearing strata of Tertiary and early Quaternary age which underlie so great a part of the Great Plains east of the Rocky Mountains. In parts of this area these strata are close enough to the surface for water to be pumped from them and often they yield water freely in commercial quantities. However, except in the bottoms where the streams have trenched themselves through the finer surface materials down to the water-bearing strata, or in locations adjacent to the streams where water from the streams is being used or can be used for irrigation

[2] Numerals in parentheses, thus: (17), refer to corresponding items in the Bibliography (see Appendix).

of the uplands, there is no opportunity for artificial recharge. Except for the negligible percolation from streams into these strata, the natural supply is the very small portion of the insufficient precipitation which is not used by vegetation, or which does not run off and thus must percolate to the ground-water body. The area irrigated in these regions outside of the stream bottoms is not large.

Alluvium Provinces Important for Ground-Water Supplies

After the foregoing exclusions, only certain unconsolidated alluviums remain for consideration in this paper. These are (*a*) the Quaternary (with some Tertiary) alluviums of the "mountain and valley" ground-water province (8)(11); (*b*) the Tertiary and Quaternary alluviums in the stream bottoms of the Great Plains, and the same alluviums beneath irrigated areas near the

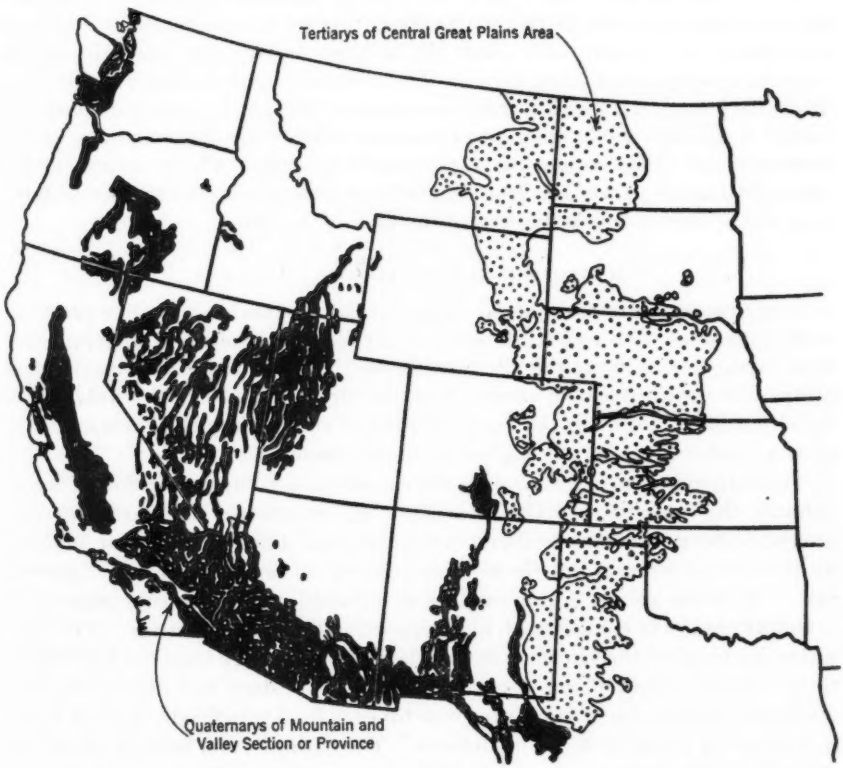


Fig. 1.—Location of Alluviums that Yield Important Ground-Water Supplies

major streams of the region. Of these the alluviums of the mountain and valley province are by far the most important for the purpose of this paper. Irrigation is practiced throughout this region and, as ground-water developments are extensive, it is believed that a greater volume of water is extracted from this

formation than from any other formation in the world. However, it is not implied that ground water can be produced in commercial quantities from all of the valleys of the province, nor even from most of them.

The mountain and valley province includes the Great Basin which embraces almost all of Nevada, western Utah, a part of southern Idaho, southeastern Oregon, and most of the southeastern desert region of California. The province also includes the coastal and central valleys of California, southern Arizona, and the Rio Grande Valley above El Paso, Tex. The portion of the Great Plains province considered herein borders the Rocky Mountains to the east and forms a wide band through North Dakota and southward almost through Texas. Fig. 1 which indicates the general locations of these sources is an adaptation of two illustrations in U. S. Geological Survey *Water-Supply Paper No. 489* (11).

The characteristic smaller valleys of the Great Basin are bolsons or semi-bolsons; that is, they either have no channel draining them and the water reaching them is disposed of by evaporation from central playas or they do have a longitudinal channel which partly drains them leaving the remainder of the water to be disposed of from playas. However, the larger streams of the basin discharge into perennial lakes and for purposes of this paper have characteristics similar to streams discharging into the ocean which drain the remainder of the mountain and valley region. In the Great Plains region all the streams under consideration are part of the Mississippi River system or of the smaller systems west of the Mississippi draining into the Gulf of Mexico.

Characteristics of Alluvial Valleys

A characteristic of the alluvial valleys of the mountain and valley province is the immense depth frequently attained by the unconsolidated alluvium, the large storage capacity of the alluvium below the water table, and its permeability. The water-yielding strata along the streams, and near-by uplands, of the Great Plains region are by contrast often of very limited thickness and thus of small water capacity although often highly permeable.

Beneath every surface stream in alluvium is an accompanying flow of water through the interstices of the alluvium. In exceptional circumstances this ground-water stream may be diverted to a direction different from that followed by the surface stream, but these circumstances do not invalidate the general rule. With the greater water supplies of a humid region this subsurface flow is almost always in contact with and supporting the surface stream. The low-water discharge of the stream is fed by the effluent seepage from the high water table. In the Great Plains area, at least in the western and drier parts, the discharge of the major streams except at times of local rain floods, in most cases, is supplied by runoff from the mountains. The natural water table of the major streams in this area is about at stream level. Generally (but not always) percolation is influent to the ground water; but only to the extent necessary to supply "evapo-transpiration" losses in the stream bottoms although there are reaches with a water table not in contact with the stream. (Effluence from the ground water to the stream when it does occur is also very small generally but a special case is the underflow—totaling annually several hundred thousand

acre-feet—from the sand hills north of the Platte River in Nebraska. This flow augments the Platte from Bridgeport to Gothenburg, Nebr., a distance of 160 miles.)

In the mountain and valley province the streams discharge in general from steep mountains either in sudden rain floods or in more gentle snow-melt runoff. The flows are often sufficiently violent to cause the tributary streams to form steep cones of coarse debris along the sides of the valley and in almost all cases cones are evident. The slope along the trough of the valley is generally less steep. The water table is often far below the surface along the sides of the valley but may be near or at the surface along the trough. Obstructions beneath the surface caused by a narrowing of the valley or by a fault zone which has been active during the period of alluviation, or merely by a decrease in the permeability of the alluvium or by other reasons, will generally bring the underflow to the surface, thus joining the surface and underground streams. The seepage at this point becomes effluent; that is, it flows into the stream from the ground water, whereas in reaches where the water table is far below the ground surface it is influent.

The storage capacity in a reach of a surface stream is small or large with reference to the discharge in that reach depending on rate of flow—the slower the movement the greater will be the relative storage capacity. Water flowing through the interstices of alluvium moves very slowly; so the storage capacity in relation to discharge past any point is astronomically large. Thus any portion of the underground stream is a storage reservoir although not strictly so in the accepted sense of the term when a surface reservoir is under consideration. From this underground reservoir water can be drawn, and into it water may be charged if the water table is lower than the stream bed.

Rising water in these streams is the ground water moving downstream which has been brought to the surface by an obstruction. Even if the obstruction is not sufficient to so reduce the cross section that the underflow is brought to the surface, it will be raised to an elevation at which pumping extraction is easier. The term "basin" is often applied to the area above a subsurface or surface obstruction. Other criteria may also lead to the use of the word "basin" but there is no general definition. The term is used very loosely, but in spite of vagueness it is a convenient one. If the alluvium is deep enough, a basin in the water table could be created at almost any point by pumping and thus lowering the water table below the elevation that will be maintained some distance above or below or to the sides of the location of the pumping.

It is probable that the water supply of none of the stream systems of the west can be fully developed without utilizing some part of the storage possibilities of the unconsolidated alluvium; nor can it be fully developed without construction and utilization of surface reservoirs. The best development will balance the advantages and disadvantages of surface and underground storage, using both in the ratio found applicable to the particular stream system and the regimen of its flow.

A study of evaporation losses from hypothetical surface reservoirs on all streams in California (16) indicates that, no matter how large the reservoir capacity, streams of erratic annual and cyclic flow will yield for useful purposes

no more than 50% or 60% of the annual average discharge because the remainder will be lost, over the years, by evaporation from the excessive water surface of the reservoir necessary to impound the water of the infrequent years of large discharge. Reservoirs of much smaller capacity would make almost as much water available. In streams of more regular regimen of flow the percentage of water that could be made available is larger; but it is evident from the aforementioned study, and it is common knowledge to those who have planned large developments, that the proper ratio of reservoir capacity to average annual stream discharge is definitely limited no matter what the extreme variations in annual discharge may be and that evaporation is one of the limiting factors.

On the other hand, if the elevation of the water table in a ground-water unit is below the root zone of overlying vegetation and below the limits of capillary action, loss by evaporation will be practically nil. Loss will occur, nevertheless, but it will be due to escape from the ground-water unit at the lower end of the unit whether as underflow or surface flow or transpiration from water-loving vegetation. Under the conditions presented in this paper, water underground is not static as it is in a surface reservoir when the water level is below the spillway lip and outlets are closed. On the contrary, it is moving as a stream or group of streams through the unit and toward the locus of escape.

Large and not directly controllable losses thus occur from both surface and underground reservoirs. For that reason water cannot be held in any reservoir for an indefinite period. Its supply must be replenished periodically and used before it is lost.

Factors that tend to make the use of ground water attractive are the gradually increasing cost of the gravity developments and the decreasing cost of pumping due to decreasing power cost and increasing efficiency of pumps. Furthermore, its use has two important advantages to the farmer—ground water does not carry weed seeds as does surface water, and it is available on demand for irrigation without waiting on rotation schedules. This immediate availability promotes over-all efficiency of farm operations although the cost of the pumped water may be greater than that of gravity water. Thus the relation between the desirability of construction and use of surface reservoirs as opposed to the use of underground reservoirs is altering progressively in favor of greater utilization of ground-water reservoirs in complete stream developments. The high prices paid for farm products is accelerating the construction of wells and the use of pumps. Once installed, the equipment will be used when desirable whether present prices for farm products continue or not.

In what follows frequent reference is made to the North Platte and Platte river basin of Colorado, Wyoming, and Nebraska, and to San Gabriel River Valley of Southern California. These two stream systems have been chosen because they involve extreme but quite typical conditions of the Great Plains province on the one hand and of the mountain and valley province on the other hand. In addition, an enormous amount of information is available about each.

In locations where utilization of the natural ground-water supply by pumping has been the paramount development (as has been particularly notable in California), it is being found that surface reservoirs must be built if

much further conservation is to be accomplished even if the only function of such reservoirs is to equate the floods so that more of the stream flow will percolate to the water table in crossing the valley or so that it can be diverted feasibly in canals to off-stream spreading grounds.

A comparison of the situation on the coastal streams of California south from San Francisco with the situation in the Platte River basin is illustrative.

Platte River Basin

In the Platte River basin, surface reservoir development on both the North and South Platte rivers (which unite in Nebraska to form the Platte River) has been so large in comparison to annual stream discharge that it does not seem feasible to extend it farther although quantities of water are lost out of the irrigated area in both streams, particularly the North Platte, and severe shortages occur in parts of the irrigated areas. The irrigation projects that are diverted from the streams by gravity have created large return flows to these same streams which have been used by downstream irrigation developments. This return flow comes from the large amount of storage artificially induced beneath the surface of the lands irrigated by gravity along and near-by the streams and is fairly uniform as compared to the regimen of the natural stream flow. Individuals or groups can pump this water while it is still in storage, for use of immediately overlying lands, and thus in times of drought make it available to themselves instead of allowing it to proceed as return flow to others downstream from them. Such pumping was begun in earnest in 1934 (the worst of the drought years) and, since then, on the South Platte and on the main Platte rivers, its progress has been accelerating rapidly. It reduces the return flow and those dependent on such flow for gravity diversions are being forced to seek other methods of securing a supply. This need has been met in part by local surface reservoir construction and in part by pumping from the ground water beneath the local irrigated area which was deprived of gravity

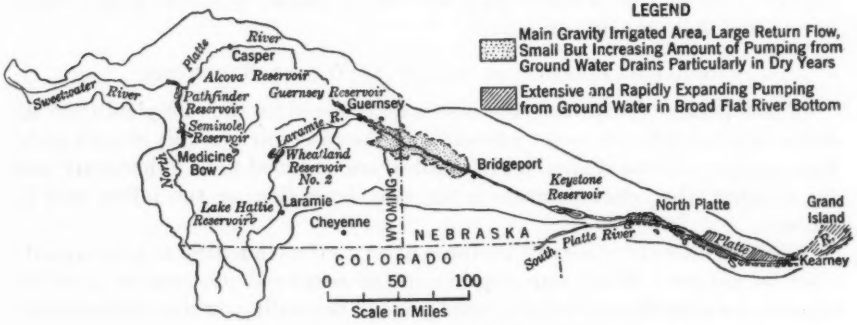


Fig. 2.—North Platte River Basin

water by the new upstream draft on the ground water. Thus pressure of need is bringing about a development whereby the utilization of water is approaching the ultimate. This may be a logical development, but it is being made without long-range advance planning. The drainage area of the North Platte River

basin is shown in Fig. 2. Since the water of the South Platte River is virtually all consumed in Colorado, the Platte River is treated as an extension of the North Platte River.

Development of ground-water use similar to that in the Platte River basin is under way in the lower Boise Valley, Idaho, and in the Rio Grande basin in Colorado. It is well advanced in Salt River Valley and Gila River basin, both in Arizona, to mention only a few. This extensive use of ground water was not planned although the early conception of Salt River Valley project did envision some use of pumped water. Such conception was lacking in other places. The developments apparently have just happened for the most part and this fact is no great compliment to engineers and hydrologists. However, the lessons learned will be of aid in the future.

In the coastal valleys of California south from San Francisco, development has been almost exclusively from ground water, resulting in some places either in cumulative overdraft or expensive pumping lifts. Reservoir development in the headwaters of Santa Clara County streams draining into southern San Francisco Bay and of Los Angeles County streams has been made or is in process, but these developments are not designed to provide gravity water to the irrigator, but rather to give greater recharge to the ground water from which the irrigator can pump it.

Thus, whether the first and the major development of any particular water supply is from surface or from underground supplies, utilization of all opportunities for water salvage within some practical limitation finally occurs, although it almost goes without saying that the percentage of salvage by surface reservoirs, in such situations as are found in the Platte River basin where large and feasible surface reservoir sites are plentiful, will always be far greater than the percentage of salvage by ground-water reservoirs. In such situations as exist in the coastal valleys of California where surface reservoir sites in the mountain drainage areas are few, of limited capacity, and costly to construct, and where ground-water reservoirs of such large capacity exist, the ground-water reservoir will always be paramount.

Characteristics of a Ground-Water Reservoir

The capacity of a surface reservoir is determined by the safe yield from the stream that supplies it, and by the cost of the reservoir per unit of such yield. The capacity and the desired useful outflow are designed to be in harmony with one another, the relation between the two depending on the inflow and its regimen.

The ground-water reservoir, on the contrary, exists in nature and its capacity is set by nature. When man appears on the scene and proceeds to upset the balance of nature the reservoir is empty or full depending on the relation of the surface stream level to the surrounding terrain. In such a situation as the North Platte Valley astride the Nebraska-Wyoming line (and of course many other valleys of the Great Plains area) the reservoir was almost empty when developments began because it was drained by the trough of the stream which is cut far below the bordering terrain and because precipitation is not sufficient to maintain a water table appreciably above stream level. In the case of San

Gabriel Valley in California (and many other valleys of the mountain and valley province), the reservoir was full because practically the only outlet is the Paso de Bartolo (Whittier Narrows) which forms a spillway lip at the southern side (see Fig. 3). The streams entering the valley do not drain the ground water but feed it, and precipitation on the valley floor is also a considerable source of supply. Development of irrigation in the North Platte Valley, which is practically all by gravity, has resulted in filling that ground-water reservoir. Development of irrigation in San Gabriel Valley, which is practically all by pumping, has resulted in partly emptying that ground-water reservoir.

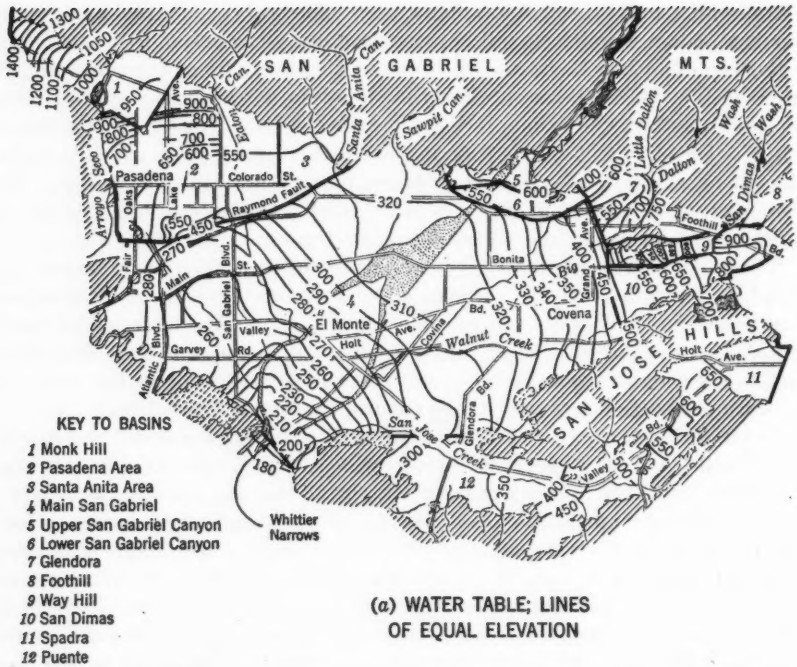
The ground-water reservoir exists in nature but its capacity is unknown and cannot be determined easily. Unlike a proposed surface reservoir its existence as such has rarely if ever been consciously envisaged until large use of water from it had developed. When pumping from it began the most important matter was whether good yields could be secured from wells at low cost. If artesian flows were discovered it was naturally so much the better.

As pumping increased and the underground reservoir was envisaged its safe yield became a most important consideration. One of the components that determines this is the capacity of the reservoir but, as before stated, its capacity cannot be measured directly as can that of a proposed surface reservoir. However, in the stage of development at which the ground-water reservoir is being consciously utilized, a mass of data will have accumulated concerning return flow, canal losses, precipitation, evaporation, consumptive use, stream-bed losses, change in water-table elevation, surface culture, stream discharge, annual pumping draft, etc. All of these items may not be applicable to any one ground-water basin but from those that are applicable, when properly assembled and interpreted, it may be possible to find the safe yield without determining capacity, as is necessary when a surface reservoir is contemplated. If such data are not available it may be necessary to estimate capacity through determination of specific yield of the alluvial material from which storage capacity may be calculated, approaching the problem much as an estimate of safe yield of a stream on which a surface reservoir is to be built is approached—that is, by determination of supply to, and demand on, the stream. This may be advisable sometimes with a ground-water unit even if all the applicable aforementioned data are available. Any such determination involves analysis of large amounts of data, but a direct determination based on recorded experience may involve less work than a determination based on capacity as an antecedent. Regardless of the work necessary the former method has merit because the result is found almost directly instead of as a residual quantity.

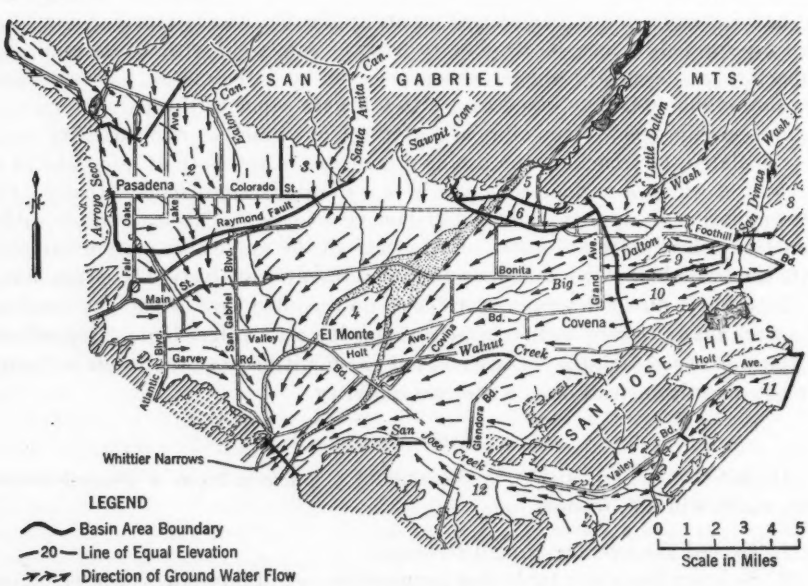
Safe Yield

Definitions.—"Safe yield" is the annual extraction from a ground-water unit which will not, or does not—

1. Exceed the average annual recharge;
2. So lower the water table that permissible cost of pumping is exceeded; or
3. So lower the water table as to permit intrusion of water of undesirable quality.



(*a*) WATER TABLE; LINES OF EQUAL ELEVATION



(*b*) GROUND WATER FLOW

FIG. 3.—OBSERVATIONS, SAN GABRIEL VALLEY, CALIFORNIA, IN THE FALL OF 1941

"Specific yield" is the ratio (expressed as a percentage) of the volume of water which may be drained out by gravity from a given volume of soil. Possibly simpler terms would be "drainable voids" or "effective porosity."

"Specific retention" is the ratio (expressed as a percentage) of the volume of water which is held against the force of gravity in a given volume of soil. Possibly a simpler and sufficiently accurate term would be "field capacity."

Determination of Safe Yield.—Disposal must equal supply. This is fundamental to all physical processes. The supply of water to a unit or reach of a stream system is the surface flow and underflow into the unit and the precipitation falling on the unit. Final disposal is by surface flow or underflow or both, by evaporation, or by artificial withdrawal. If final disposal is in temporary imbalance with supply, the storage in the reach of the stream changes. The statement applies both to the part of the stream underground and to the portion on the surface.

Over a term of years the natural escape of ground water from a unit balances the natural supply to the unit. The amount escaping at any particular time is governed by the elevation of water table. In many cases the supply is also modified by this because the natural water table may be so high as to preclude percolation from parts of the streams. When pumping is begun the new and additional draft causes the water table to lower and natural outflow decreases. At the same time the lowered water table may induce a greater supply because of greater percolation opportunity. In other words, any reservoir, when half full, can and will impound more of the water that reaches it than it could if it were three fourths full or entirely full. Also, discharge through the uncontrolled outlets of a reservoir or over the spillway decreases as the water level in the reservoir decreases.

As a physical matter only, the pumping draft might become so large that outflow would be stopped. This possibility would depend on the slope of water table toward the outlet and on the depth of alluvium, but as a practical matter such lowering of the water table would probably bring about higher cost for the pumped water than would be feasible. Therefore, there is a financial as well as a physical limit to the safe yield of a ground-water unit. The essential problem of safe yield from a ground-water unit is to determine that part of the natural disposal that can be or has been diverted to beneficial use by pumping from the basin and to what extent the recharge will be or has been increased by development.

If the water is to be used on overlying land the salts in the return flow finally concentrated in the ground water would make it inexpedient to use it all. Some escape would be necessary to maintain the salt balance, for a ground-water unit needs "sewage" disposal as imperatively as does a municipality.

Thus there are two limitations on the safe yield as it might be calculated from consideration of only the supply to the unit and the void space in the alluvium. One of these is cost and the other is quality of water. However, a start must be made and the limitations due to cost and water quality may be considered after the physical possibilities have been explored.

It was stated previously that by the time the stage of development has been

reached at which the ground-water basin is consciously treated as a ground-water reservoir, numerous data are available as to various phenomena that will aid in determination of safe yield. The question arises as to whether it would be possible to determine safe yield of a virgin ground-water basin. The quantities of water percolating to the ground-water basin or leaving it as transpiration, evaporation, and rising water would be the only criteria; but precipitation varies cyclically (to use a not wholly accurate but quite descriptive term) as well as annually and in the arid regions of the United States the cycle may be long and the percentage of variation in the cycle large. In Southern California the cycles during the period of record have been from 20 to 30 years long. If there had been no development in San Gabriel Valley and if an observer had measured the rising water at Paso de Bartolo for the period from 1905 to 1933 he would have been able to calculate the average surface outflow from the ground-water basin above. If he had been informed as to depth of the alluvium in the valley he would have known that sufficient capacity exists to give a safe yield equal to the average yield. However, he would not have known that the safe yield would be larger than the surface outflow because of greater recharge due to change from native to cultivated vegetation and to lowered water table which gives greater percolation opportunity. At any rate, only long-continued observation would have yielded satisfactory information. (The discharge of rising water from the valley varies with the elevation of the water table in the central part. Consistent measurements of rising water which began in 1923 and measurements of elevation of water table in the central part of the valley which go back to 1903 indicate that a fairly definite relation exists. The highest elevation was recorded in 1916 and the lowest in 1934. The highest average discharge of rising water taken from the relationship curve was 165 cu ft per sec in the water year 1915–1916 and the lowest (measured) was 38 cu ft per sec in 1933–1934.)

From some valleys, particularly those of the Great Basin, there is no surface drainage and the water supply is disposed of by evaporation from playas and transpiration from vegetation whose roots feed on the ground water near the low point of the valley. The area of high water table must vary with the climatic cycle and, although the draft on ground water per unit of area of such vegetation can be closely approximated by appropriate methods, it would seem that changes in the area of vegetation drawing on ground water would occur, thus making it necessary to wait years before this would be a definite measure of the safe yield.

It may be concluded that seldom, if ever, can safe yield be determined with any great accuracy until the data available from more or less haphazard development are at hand; but until such time comes probably approximations can be made by applying factors to a limited amount of data. However, only infrequently, if at all, would there be occasion to determine the yield of a virgin basin.

The Hydraulic Equation.—The various elements of disposal and supply for a stream in which a ground-water reservoir, basin, or unit is involved are the

same as for a unit of a surface stream. The equation is as follows:

Surface outflow		Surface inflow
plus		
Underflow out		plus
plus		
Consumptive use	equals	Underflow in
plus		
Change in storage		plus
plus		
Exportation		Precipitation on surface

The equation is general but there are refinements which must be used under certain conditions.

Surface Outflow.—The lower end of a ground-water unit is generally at a geological formation either of relatively impervious alluvium or at a narrows which is a strategic location for a stream gaging station.

Surface Inflow.—The upper end of a ground-water unit is likewise generally at a strategic location for a stream gaging station. Strategic locations are generally available on tributaries also above the ground-water basin, but, if unimportant, their discharge can be estimated with sufficient accuracy.

Underflow.—Measurement of the outflow underground has been tried in various places with little success except in those cases in which its amount is negligible. As the underflow is generally small when compared to other quantities, because the area of permeable alluvium is constricted at points of outflow, a considerable error in the estimate of this item may not be important in the over-all result. Often the only inflow underground is the probably negligible amount from the rocks of the basement complex of the region.

Consumptive Use.—Much work has been done on consumptive use of individual crops and types of native vegetation by H. F. Blaney, M. Am. Soc. C. E., and associates (23)(26)(28)(29)(30)(31)(35)(39) and on total consumptive use of valleys as a whole. Therefore, it is possible to assign values to this item based on experience elsewhere, properly modified for variations in temperature and surface culture, or based on data from the unit under consideration itself (1)(4).

Change in Storage.—In all localities where ground water is important, interest has turned to measurement of depth to water from the surface at wells. As the years have passed it has come about that such measurements are generally made systematically and thoroughly. It is not always possible to determine whether a measurement discloses the distance from the surface to a column of water under pressure or whether it discloses the distance to a free water table. Consistency of measurements, study of results, and in some cases installation of recorders have helped in separating the two but difficulties will always be encountered. Changes in piezometric level, of course, do not indicate change in storage. For a pressure area the change in elevation of water table at the forebay (if such exists) is the indicator. These measurements give the change in volume of saturated alluvium which has taken place in a stated period of time and are thus indexes of change in amount of water stored.

Change in storage in an underground reservoir is the integration of the values of all the divergent forces acting on the ground-water basin. It includes the effect of the unknown underflows and, in fact, none of the other quantities noted in the equation except extractions need to be known for the determination of safe yield for any stated period. ("Safe yield" may be used as meaning the volume of water pumped or the net volume after consideration of return flow and other new recharge, which is lost to the ground-water unit due to pumpage. In this paragraph the former meaning is in mind. In the hydraulic equation the latter meaning is embraced in the terms "consumptive use" and "exportation." Consumptive use of pumped water is meant.) As compared to most of the other quantities and especially to the sum total, change in storage can be expected to be small. Even at the most it is unlikely that it would be more than 10% of the difference between total water entering (including precipitation) and leaving the unit and it will usually be far less. Error in this one quantity, even if in the same percentage order as the errors in other quantities involved, would therefore ordinarily be only a small part of the absolute errors involved in the other quantities.

Disregarding determination of underflow, determination of change of storage capacity via an antecedent determination of specific yield is the most difficult of all the items of the equation but is evidently the most important in those cases where such determination is desirable. It may be possible in many cases to estimate it from general physical conditions with sufficient accuracy for preliminary purposes and thus avoid elaborate determinations. Short cuts along this line have been evolved. The growing knowledge of values for consumptive use of vegetation is giving valuable aid.

Exportation.—Water used on overlying land gives return flow to the basin unless continuous impervious strata intervene between the surface and the aquifer. Exportations of pumped water do not give a return flow, and therefore the two must be kept separate in the hydraulic equation.

Precipitation.—In some regions fairly distant precipitation records are satisfactory. In others, notably the mountain and valley province, and particularly in the coastal valleys of California, variation is great in a few miles even on the valley floor and numerous records are necessary.

Observed General Values of Certain Phenomena.—It will be observed that the general hydraulic equation will give an accounting of the disposal of the supply for any stated period and involves both the surface flow and underflow of the reach of the stream in question. It does not necessarily provide a complete basis for extending experience into the future—that is, for the design of a project (if the term may be used), or for safeguarding prior rights dependent on the natural percolation. To do this it may be necessary to secure data on the variation in amount of percolation with variation in discharge of the streams, with variation in annual precipitation, with anticipated changes in surface culture, and with anticipated changes in elevation of water table.

In such case the hydraulic equation refers to supply to, and disposal from, the ground-water unit itself instead of from the entire portion of the stream system which contains the ground-water unit. The members of the left-hand side of the equation would be unaltered but "consumptive use," instead of being

that of the entire basin, including that from precipitation direct, is only that which draws on the ground water; that is, the use by the area irrigated by pumping and by water-loving vegetation where the water table is high. "Surface inflow" on the right-hand side of the hydraulic equation becomes "percolation from stream flow," and "precipitation on surface" becomes "percolation of precipitation."

The determination of these quantities is much more difficult than the recording of precipitation and runoff since only the portion of these quantities which percolates is involved. These various items of recharge can be discussed only generally in this paper.

Percolation from Precipitation.—The portion of the rainfall on the alluvium of the valley floors which percolates past the root zone of vegetation is a supply to the underlying ground-water basin unless continuous impervious strata intervene. Average annual precipitation in the valleys of western United States varies from about 2 in. to about 30 in. East of the Rockies about 65% of the total falls in the late spring and summer, whereas on the Pacific Coast about 75% falls in the winter. In other areas the distribution through the year is more nearly uniform. Probably the runoff from any of these annual average rainfalls is not more than 5% in agricultural areas. The amount that gets through the root zone and thence to the ground water depends on the regimen of precipitation, the type of vegetation, the type of soil, and the temperature. The native vegetation tends to be of the highest type that the precipitation will support and hence consumes almost all the average rainfall even in a subhumid climate. Thus, percolation to the ground water in areas where the surface is covered with native vegetation probably occurs only in the years of considerably above-normal precipitation. A rough estimate would be that in such areas practically no precipitation escapes unless the annual average is above 15 in. and that with an annual average precipitation of 30 in. the average annual contribution to ground water is generally about 2 in., but that it varies from less than 1 in. to as much as 5 in. in exceptional circumstances.

In order to capture the rainfall the native vegetation often must develop tremendous root systems. These are lateral systems in extremely arid regions where depth of penetration of precipitation is necessarily very small and for this reason the individual plants are widely separated. On the other hand, where the soil is coarse and where precipitation is great enough to penetrate deeply, especially if it occurs in the non-growing season and thus must be stored in the soil because it is not consumed immediately, the root systems do not have disproportionately large lateral spread. Rather, they extend to great depths in order to use the greater part of that which has penetrated beneath the surface and been stored. As most cultivated vegetation except alfalfa, deciduous trees, and vines, has a comparatively shallow root system a larger amount of precipitation goes through to the water table after land in the less arid regions with the Pacific type regimen of precipitation has been placed in commercial crops.

Annual average precipitation on the well-developed Raymond basin area northeast of Los Angeles, Calif., is 22 in., of which 75% falls in the four winter months. Here are found the City of Pasadena, Calif., and several other cities,

as well as irrigated groves (41). The average annual supply to ground water of this area from precipitation direct is about 5.3 in. in depth. When the area was in a virgin state, the deep percolation was about 2.0 in. Precipitation on the valley floor accounts for about 44% of total supply to the ground water. About 27% of the surface is impervious because of paving, sidewalks, and buildings. The culture on another 48% has been changed from the native state but change in root zone depth on this part has been immaterial. The major increase in penetration of rainfall to the water table has been from the remaining 25% of the area and the total amount of such increase is striking in the extreme when it is realized that it comes from so small a part of the area. In this part average annual penetration of precipitation is perhaps about 13 in., or six times as much as when it was in deep-rooted native vegetation. This increase, however, is partly due to runoff from impervious roofs of residential buildings to the pervious shallow-rooted areas of lawn and garden.

Such calculation of percolation from precipitation separate from irrigation water might not be possible where a large part of the precipitation occurs in the summer coincident with irrigation; nor in most cases would it be necessary. In such cases the precipitation is added to the irrigation water diverted to the area and the combined total is used in general calculations. Due account must be taken of increased over-all consumptive use due to almost total evaporation of small rainfalls.

After irrigation development takes place it would seem probable that in regions where precipitation occurs in the growing season contributions would occasionally occur to the ground water even with annual average precipitation of from 7 in. to 8 in. However, in a large part of the Great Plains region the soil is close textured and its field capacity is large. Although irrigation is practiced for general crops where average annual rainfall is as much as 24 in., precipitation in that region occurs mainly in the growing season when transpiration is heavy and it does not penetrate deeply; nor are deep-root systems necessary to take advantage of it inasmuch as it is used up very soon after falling and there is little accumulation. Hence contribution to ground water from precipitation on such soils is small even after the surface culture changes from native to cultivated types. Actually in this region (except in coarse soils such as the sand hills of western Nebraska, where the average annual rainfall is about 20 in.) the contribution to ground water from precipitation is almost negligible and exploitation may quickly cause overdraft.

Percolation from Stream Flow.—Under natural conditions and in regions of less than about 15 in. average annual precipitation, percolation from streams supplies practically all of the natural ground water of the valleys in the mountain and valley province. The water table is generally far below the surface and the material of the stream beds is generally very receptive of water. Percolation is greater in that portion of the stream beds consisting of coarse sands than it is in the portions wherein larger particles, termed gravel and boulders, are found because these particles occupy a considerable portion of the mass, whereas the percolate can travel only through the interstices of the sand which fills the space between the larger particles. In the San Gabriel River bed the material changes rapidly from coarse gravel, boulders, and sand near the

mountains to rather coarse sand about six miles below. Farther downstream the sand is finer.

Percolation rates from this stream measured by the State Division of Water Resources varied from 0.90 cu ft per sec per acre near the mountains to 4.84 cu ft per sec per acre at about 6 miles below for the first few days of flood discharge (18)(20). From that point downstream, measured percolation rates decrease gradually to about 1 cu ft per sec per acre in the next two miles and presumably the rate is smaller farther downstream. A feature of percolation in this vicinity is its rapid decrease in the portion of the river bed near the mountains as the flow recedes and becomes more uniform after a flood. This is because of the deposit of calcareous material which forms on the stream bed near the mountains. In these reaches the gravel and boulders are of sufficient size to be undisturbed by the smaller flows. Downstream from this coarser material the lighter sands are moved continually by flows which do not disturb the heavier material and the calcareous coating is not formed until the flow gets smaller.

By a series of intensive measurements on San Gabriel River during flood times and subsequently until water ceased to flow into the ocean, a formula was established which has been expressed on a diagram (20) giving the volume of water which will percolate for different discharges at the mouth of the canyon from which the waters of the river debouch on to the valley. This diagram is somewhat similar to a rating curve for a stream measuring station but is more complicated.

A similar series of measurements and determination of percolation on Alameda Creek tributary to San Francisco Bay has also been made. The formulas derived apply only when the condition of the stream bottoms is as it was at the time of the investigations. The conditions in many of the coastal streams of California are similar to those of the San Gabriel River, and the formula applicable to the San Gabriel River has been used in calculations of loss from these other streams in the belief that substantial error would not result even if the ratio of percolation to discharge on these other streams was considerably different, which seems unlikely. This seems permissible and especially so because the number of days of flood flow in which the streams persist to the ocean is comparatively few. At other times the entire stream percolates and the cumulative total of percolation at such times is much more than the cumulative total during flood times.

Ordinarily, stream-flow measurements to determine percolation are too expensive to be practical except at low flows but, in these cases, were believed to be necessary for protection of prior rights, inasmuch as proposed reservoirs would disturb the flows and the percolation.

Percolation from Water Spread on Surface (5).—In Southern California at about the turn of the century, the practice was introduced of diverting the water of a stream over waste land below the canyon mouths where the water table was distant below the surface thus causing it to cover a greater area and giving it a greater opportunity to percolate. This is much the same as irrigating the land by wild flooding although for a different purpose than ordinary irrigation. Water is also caused to occupy a large area of the stream bed

itself for the same purpose. In some cases the water diverted from the stream is ponded behind low dams or rock walls. The practice has since been adopted in other states.

Unless the stream is regulated by a reservoir which will also desilt the water, spreading cannot be done in times of violent rain floods because of the hazard or because the silty water will seal the surface of the gravel and decrease the percolation rate. Therefore, spreading on unregulated streams is usually done at other than flood times, and most of the discharge in many such times would percolate even if not spread (because of the small volume), although at some lower point on the stream.

The more uniform snow-melt floods carry less silt and fewer precautions are necessary. Spreading other than small quantities on unregulated streams may be quite a costly matter when net benefits are considered. Published data are misleading in that they give gross amounts diverted and spread without reference to the amount which would have percolated anyway. Often too, published data are not actual records of even gross amounts diverted.

Spreading from regulated streams is more successful and possibly will be ultimately developed in many streams where conditions are favorable. The surface reservoir regulates the flow so that it can be handled more easily. Spreading under such conditions is equivalent to a transfer of water from a surface on-stream reservoir to a surface off-stream reservoir, in this case represented by the underground reservoir. Even in such cases published records may give exaggerated values.

The initial rate of percolation from spreading grounds may gradually decrease through each spreading season and the average rate for each successive spreading season may decrease until spreading is no longer possible. This phenomenon may be due to changes in the soil caused by base exchanges. Other possible causes exist. This deterioration appears to be less evident, or entirely lacking, on debris cones of coarse material where native vegetation is undisturbed.

Water is sometimes diverted to wells which are not being used and the water is forced into the gravels by the head in the well thus created. The amounts thus caused to percolate have been comparatively insignificant and in some regions, due to deposition on the sides, the possible future utility of the well may have been impaired. However, in regions where deposition would not occur, it would appear possible to recharge ground water through wells in situations where a distribution system is installed having surplus capacity and surplus gravity water at a coincident period. Such recharge would seem to be of special importance in connection with demands on ground water for air conditioning now expanding so rapidly and also for general municipal drafts because such distribution systems lend themselves more readily to the practice and the draft is more localized than in the situations under discussion here. The matter is somewhat outside the scope of this paper and is here mentioned only in passing.

An important consideration often overlooked in evaluation of the benefit of spreading to induce additional underground supply is as follows: Unless the draft on the ground water of the basin into which the water is caused to perco-

late is greater than the recharge the water caused to percolate will be lost by increased transpiration from water-loving vegetation near the lower side of the basin or by escape as increased surface outflow or underflow. The sum of these increases would equal the additional water caused to percolate. The benefit from spreading in such basins is limited to the smaller pumping cost due to higher water table which exists primarily in the drought phase of the precipitation cycle inasmuch as, during the surplus phase, the water table would raise from the natural supply to about the same elevation as it would with the additional induced percolation. This benefit must be balanced against cost of spreading and loss of water to the stream due to increased transpiration in areas of high water table which reduces the supply to other users below.

Percolation from Gravity Irrigation.—Whenever water is used for irrigation the surplus beyond the amount consumed or returned directly to a water course percolates to the ground water. This deep percolation may be small or large depending on soil, crops, cost of water, etc., but it is inevitable. If the water is from a stream, thus constituting a new supply, the percolate finally fills the alluvium beneath to the level at which it will be disposed of in obedience with physical law. The disposal will be accomplished by evaporation if the topography is such that the water table approaches close to, or intercepts, the ground surface or by seepage into or by discharge of springs into drainage channels. Such discharge is called "return flow." It is the escape from the ground-water reservoir beneath the irrigated area which has been filled by percolation of water diverted from a stream and by any additional percolation of precipitation due to irrigation.

These irrigation enterprises accomplish much the same thing as "spreading" to induce percolation, but actually the amount of percolation is generally much larger although in most cases only an incident to direct beneficial use. Possibly the first conscious use of the alluvium as a water reservoir came about because of such deep percolation. This was so in San Joaquin Valley (California) where, in many parts, the excess water does not readily escape as return flow but swamps the land instead. Ground water was pumped through necessity near the beginning of the century, and, during subsequent dry periods which brought about recession of water table, water was diverted and placed upon the land during the non-growing season, or excess quantities were diverted during the growing season, in order to augment the ground water.

Specific Yield.—If the specific yield can be determined the water capacity of the underground reservoir can be calculated. Changes in elevation of water table registered in feet can be interpreted into changes in quantity of water—that is, into additions to and subtractions from the amount of water in storage. If extractions by pumping, specific yield, and change in water-table elevation are known, the safe yield for any stated period in which data are available can be determined. This information for a shorter period must be modified to find the long-time average safe yield.

In 1923 Mr. Meinzer listed seven methods for determining specific yield (11) and in 1932 he again listed the identical methods (13). In 1937 C. F. Tolman relisted them (8). It is safe to say that all direct determinations that have been made had one or more of the seven methods as a basis.

As given by Mr. Meinzer (headings are by the writer) the seven methods are:

A. Direct Determinations of Specific Yield:
1. Laboratory saturation and drainage of columns of alluvium;
2. Field saturation and drainage of alluvium as it lies in place;
3. Determining from samples the specific retention and porosity after water has been drained in field;
4. Pumping from ground water, exporting or using the water and observing the volume of alluvium unwatered; and
5. Observing the volume of water entering alluvium and the corresponding volume of the alluvium in which a rise in water table occurs.

B. Indirect Laboratory Determinations of Specific Yield:
6. By the centrifuge; and
7. Mechanical analysis and determinations of porosity, estimating therefrom the specific retention and specific yield.

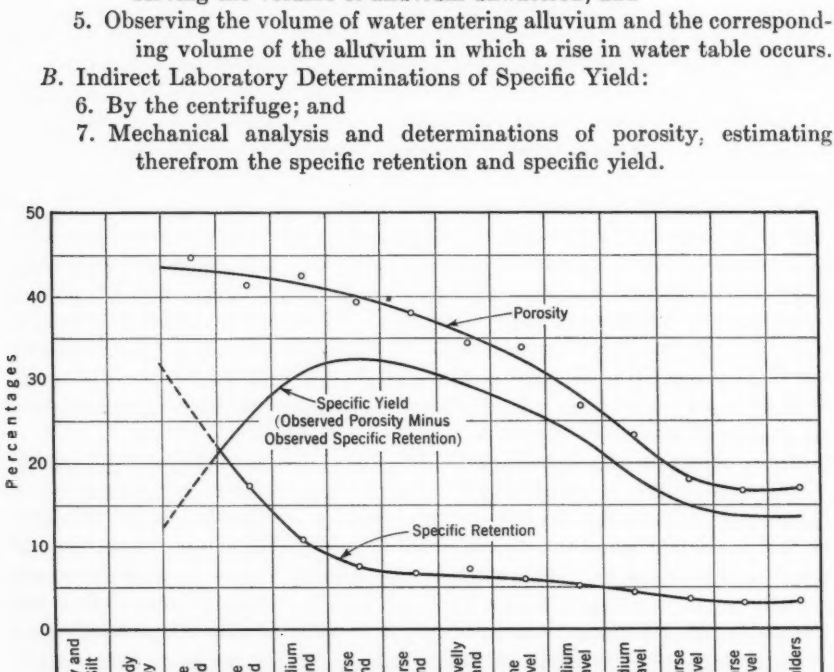


FIG. 4.—POROSITY, SPECIFIC RETENTION, AND SPECIFIC YIELD FOR SEDIMENTS OF THE SOUTH COASTAL BASIN

Adaptation in California.—Items 1 and 7, combined with much field work, are the basis of methods used for estimating safe yield in a large part of the coastal valleys of California south from San Francisco, by the State Division of Water Resources. In these valleys the water table is generally far below the surface. Supply to, and draft on, the ground water vary widely from year to year and data necessary to direct methods of determining safe yield are sometimes difficult to secure; or the methods may not be applicable in a large part of the area. In such cases specific yield must be known as an antecedent. In parts of the valleys, however, none of the aforementioned methods are appli-

cable and other more roundabout procedures have been adopted although they are less satisfactory.

Laboratory methods naturally involve the application of the determination of specific yield at several points to the specific yield of a large mass and the procedure is open to objection from that standpoint; but this same sampling procedure is followed in determinations of many other physical properties. It happens that, in determining specific yield, it is possible to take advantage of other well-known physical phenomena for applying data secured at one point to the larger area. The result is to decrease materially the amount of work necessary, and to increase the accuracy. After several years of intensive work in the South Coastal Basin of California, notably in San Gabriel and Santa Ana river basins, procedures were evolved which may be applicable to the alluvium in other portions of the mountain and valley province where conditions are similar to those in coastal California (26).

A feature of this work was the objective of determining whether the slope of the debris cones formed by the streams could be used as a criterion for determining the variation in specific yield of unaltered surface material along radial lines from apex to distal portion, and whether the specific yield of unaltered surface material could be used as a criterion for the yield of altered similar material beneath the surface. It was thought that, if such relations did exist, a short cut would be available which could be used elsewhere in similar valleys under investigation with a minimum of field and laboratory work.

The study therefore involved laboratory determinations of porosity and specific retention of unaltered materials encountered in cut banks or well pits near the surface and at depth. It involved similar determinations of size of material at various distances from the mountains for streams of various sizes and various slopes of the surface. Fig. 4 shows the porosity and specific retention of a range of sands and gravels as determined by tests in the laboratory of more than 200 samples of unaltered material taken from near the surface (27). The curve labeled "specific yield" is the result of subtracting values in the "specific retention" curve from values in the "porosity" curve. It will be noted that even the finest of these materials is sufficiently permeable to yield water freely. As an easy method of determining the coarseness of the sample in the field the "maximum 10% grade size" is used. This is the grade size in which the cumulative total, beginning with the coarsest material, reaches 10% of the total sample and in these alluviums it is generally the next to largest grade size or sometimes the grade size next smaller. The maximum 10% grade size is the exact opposite of the "effective size" used by the late Allen Hazen, M. Am. Soc. C. E., in his investigations on sand filters which is the grade size in which the cumulative total beginning with the finest material reaches 10% of the total sample. For the entire South Coastal Basin a map showing lines of equal specific yield of surface materials was drawn and the part of it for San Gabriel Valley is shown in Fig. 5(*a*). Drillers' logs were studied and materials from wells in process of being drilled were tested so that it was found that certain expressions used by drillers such as "gravelly clay," for instance, referred to a material altered by decay from its original characteristics to a specific yield

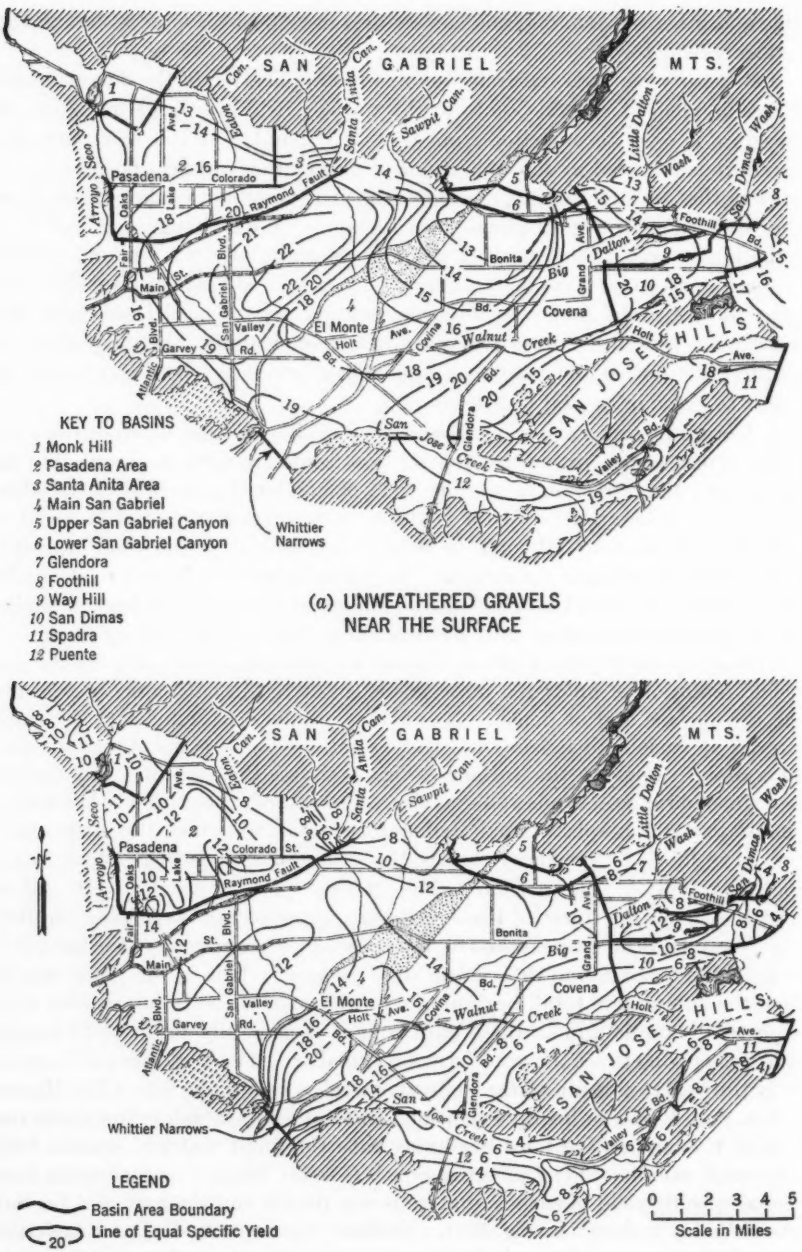


FIG. 5.—LINES OF EQUAL SPECIFIC YIELD, SAN GABRIEL VALLEY, CALIFORNIA

about one third that of the original material at or near the surface. Well logs were studied by groups and the specific yield for the group was placed at a point on the map at approximately the center of gravity of the well locations. Other criteria, such as decrease in porosity due to depth, and the steepness of cone, etc., were given weight. All this was used in drawing contours of equal specific yield for the material within the probable range of fluctuation of water table (see Fig. 5(*b*)).

It was found that distance from the mountains and the steepness of slope of the debris cone were useful as indicators of specific yield. The method resulting from this intensive study has been applied to some extent with a very small amount of laboratory work to other similar valleys in California. The results have not been tested by other means in those valleys. In the central San Gabriel Valley and in the Raymond basin area of San Gabriel Valley from which many of the samples analyzed were secured and in which much of the work was done, studies have indicated that the values for specific yield thus determined in those locations are sufficiently reliable. This method is well suited to a large part of the coastal valleys of the state and should be also suited to much of the entire mountain and valley province.

Direct Methods of Safe Yield Determination

The foregoing involves measurement of capacity for water storage between different elevations of the water table of the unit involved and this is analogous to a survey made to determine capacity of a surface reservoir. In some cases operation of a surface reservoir has shown that the surveys made to determine its capacity were in error and that actual capacity was considerably different. This has been indicated by the fact that, after allowing for losses and inflow, release of a certain quantity of water did not produce the change in elevation of the water table which, according to the surveys, it should have. In such a case safe yield can be determined from operational data and capacity can be ignored. In other words, safe yield may be determined directly without knowing capacity.

Similarly with underground reservoirs: Given a set of conditions which comply with the requirements, safe yield and other desired information can be determined. There are three variables concerned—supply, draft, and change in water-table elevation. The requirement is that one variable must be sufficiently uniform to be considered constant, thus leaving only two. The most convenient method is graphic by platting on rectangular coordinates.

Two cases will be examined by way of illustration:

Case I. Supply to the Ground-Water Unit Is Uniform Year by Year, or Period by Period.—The variables are (*a*) the annual change in water-table elevation, and (*b*) the annual draft.

Case II. Draft on the Supply of a Unit, Whether Surface or Underground, Is Uniform Year by Year, or Period by Period.—The variables in this case are (*a*) the annual changes in water-table elevation, and (*b*) the annual differences in the quantity of surface water retained in the unit, as found by measuring inflow and outflow.

In either case, even if the period of record includes only years in which the water table fell, or only years in which it rose, the points when platted will define the slope of a straight line which, prolonged, will intercept the line of zero change in water-table elevation. This value on the axis on which volume of water is platted is the safe yield in Case I. In Case II it is the quantity required to supply the demand, whereas the quantity retained as determined by the measurements of inflow and outflow is the safe yield.

For Case I the net supply to the ground water might be sufficiently uniform to be so assumed (*a*) under an irrigation project in a fairly arid country, (*b*) in an area where the supply is by lateral percolation from ground water at higher elevation, and (*c*) in an area where, even if the water reaching the overlying surface is variable from year to year, distance to the water table is so great and perviousness so small that annual variations in percolation of the surface supply are eliminated to a large extent. In general it could not be used in a region of fairly large precipitation during the non-growing season although special circumstances might make it applicable in limited areas in such regions.

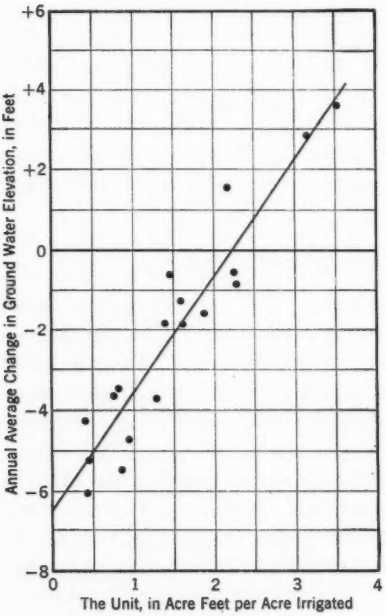


FIG. 6.—RELATION OF ANNUAL SURFACE INFLOW RETAINED TO CHANGE IN AVERAGE WATER-TABLE ELEVATION, TULE RIVER-DEER CREEK AREA

For Case II the net draft on the general water supply can be considered uniform in a unit where the irrigated area is fairly constant from year to year, and where deficiency in surface flow can be made up by pumping from the ground water. If precipitation on the surface of the unit were large and occurred in the non-growing season it might not be possible to use this procedure because of the great variability in apparent and actual consumptive use from year to year.

So far as is known to the writer, the Case I method was first used by Raymond A. Hill, M. Am. Soc. C. E., in work in Arizona. Similarly, the Case II method in this form was first used by S. T. Harding, M. Am. Soc. C. E. (17), in San Joaquin Valley and was used by the State Division of Water Resources in its studies for Central Valley Project. In both of these regions precipitation is small on the valley floors.

Fig. 6 illustrates Case II. It shows the surface inflow minus outflow per acre irrigated each year, platted against the average change in water-table elevation for that year, for the Tule River-Deer Creek area, which is one of the ground-water units of southern San Joaquin Valley (3)(8). The surface supply

is from the two streams mentioned which head in the Sierra Nevada mountain range in eastern California. The flow is diverted by gravity and when deficient is augmented by pumping from ground water which is supplied by percolation from the streams from irrigated lands.

Underflow, either in or out, is unknown and is believed to be negligible; at least the difference is believed negligible. However, this is immaterial because the change in water table is the integration of all the forces acting on the ground-water supply. The net surface supply was sufficient in those years in which no change in water table occurred to supply the consumptive use plus or minus the net underflow.

The average area irrigated in the Tule River-Deer Creek area during the 18-yr period from 1921 to 1939, inclusive, was 67,800 acres and the maximum variation from this was about 10% plus or minus. The net average annual stream flow retained was 103,500 acre-ft or 1.53 acre-ft per acre irrigated. This is the safe yield for the 18-yr period. If 2.22 acre-ft per acre had been retained, the water-table elevation would have remained constant indicating that 2.22 acre-ft per acre (plus or minus any difference between underflow, in or out) is the net requirement or a total of 150,500 acre-ft for the 67,800 acres. Therefore, the average annual overdraft on the ground water of the unit for the 18-yr period is 47,000 acre-ft. The gross area of the unit is 239,000 acres and the water table lowered an annual average of 2.06 ft during the 18-yr period. Thus the soil volume drained was 492,000 acre-ft per yr and as the overdraft was 47,000 acre-ft the specific yield of the alluvium is 9.5%.

Whether the long-time average safe yield is 103,500 acre-ft is not disclosed by analysis of these data. To determine it would require a comparison of long-time average net surface inflow with that which occurred during the 18-yr period. A similar adjustment would be necessary with any other method. To make such adjustment would require a detailed analysis of the amount of percolation reaching the ground water from stream flow and precipitation during a long-time average period of precipitation to estimate the difference between that amount and the amount which did reach the ground water during the 18-yr period. Consideration would also be given to the changed amounts of underflow in and out which might occur with a changed elevation of water table.

In southern San Joaquin Valley the precipitation occurs in the winter and the annual average varies from 5 in. to 10 in. according to location. The direct contribution from precipitation to ground water may be regarded as negligible since the area with precipitation approaching 10 in. is small. Annual changes in water-table elevation at many wells in the various cones of the major streams or ground-water units have been recorded since the early 1920's and the average for the period of record can be calculated. Surface stream discharge into and out of each unit has been recorded. Underflow either in or out is probably negligible. The acreage irrigated each year is known approximately. In the particular unit under discussion, and in some other units, it has been fairly static for many years past.

In many situations large annual variations in supply or demand may be ironed out by using moving averages embracing sequences of several years. If this occurs it may be found that the methods of Case I and Case II will be

applicable to the solution in particular areas, although not in regions of long swings in the precipitation cycle except perhaps in limited areas.

Dissolved Solids—Quality of Water

It is now known that not all the water of a stream system can be consumed in irrigation because the gradually increasing concentration of dissolved solids in it causes it finally to become so toxic to plant life that it cannot be used. A certain percentage of the water of a stream must be used to carry off the toxic material and to maintain a salt balance.

About 2500 years ago the great empires built upon the large irrigation developments of the Tigris and Euphrates river valleys deteriorated. It is surmised by some that these ancient empires retrograded because of lack of knowledge about the damage by dissolved salts to plant life; but, be that as it may, even as late as 1900 engineers in the United States knew so little about the matter that some irrigation developments (even in the part of the country where the geological formations are such that the moving water gathers large quantities of various salts) were designed without this difficulty in mind.

The problem is much less serious in some parts of the West than in others. Probably it is worst in west Texas, New Mexico, southwest Colorado, southwest Utah, Arizona, and the southeastern part of California. In some streams in other western regions there is little evidence of such toxicity, but the matter is one of degree. Water will dissolve some salts from all formations and, if the final residue of water is small enough, the solids will be sufficiently concentrated in it to be detrimental.

The same difficulty occurs with ground-water developments as with surface water but the likelihood exists, in the case of some ground-water developments, that the final disposal must be by pumping and exportation of sufficient water to carry off the dissolved solids.

Ground water suffers also in this respect from causes of concentration of solids which are not likely to exist in surface waters. Three examples may be cited:

1. Near the ocean beaches or other salt-water bodies, salinity may encroach into the pumping field if the water table is lowered sufficiently to reverse the normal gradient. This is not greatly different from the situation found in fresh-water streams for it is true also that salt water may encroach into such streams at their mouths and deteriorate the quality of the water in the lower reaches of the stream.

2. Well casings may be perforated incorrectly or they may be imperfectly set, allowing water with high concentrations of solids from a perched water table or other source to flow into the strata from which the wells of a field are drawing.

3. The ground-water basin was at one time a part of the ocean. Salt water was trapped in it and there remains below the fresh water because of its greater weight. As the weight of fresh water on it is lightened by pumping from it, and especially at the cones of depression and drawdown due to such

pumping, the salt and fresh waters tend to be mingled and difficulty occurs (2)(25).

Once the aquifers of a ground-water unit are impregnated with salt water it may be years, even with cessation of pumping, before the quality of water will be restored.

Movement of Ground Water

The rate of forward movement or flow of ground water would seldom be important to the purpose of this paper, because in most cases pumping is widely distributed over the surface of the ground-water unit and except in pressure areas the recharge is from the immediate surface. If the recharge is not predominantly vertical it may be possible to use the direct determination of safe yield by Case I without knowledge as to transmissibility. However if the recharge is largely lateral and if a part of the supply comes from a source in which concentration of dissolved solids is high (such as might be the case with a ground-water unit along an ocean beach) it would be found desirable to determine only the amount of the fresh-water supply. This could be done either by calculating the rate of movement of such supply into the ground-water unit (the cross section of the flow being known) or by calculating by other means the volume which must be entering the unit. Either method leaves much to be desired.

One important aid in determining direction of movement and influence of any one source of supply in any particular part of a ground-water unit is often the dissolved solids in the ground water and this matter should be given consideration in all ground-water studies (2)(25). However, the importance of this decreases as the slope of the water table steepens. If there are no discontinuities in the water table its slope should be a more definite indicator of direction of movement than the solid constituents of the water and their proportions. It would not, however, tell the amounts of water from each of two or more sources which reach the area of extraction. This would be best answered by determination of the dissolved solids.

Summary

This paper has sought to accentuate the fact that ground water is merely water beneath the earth's surface and that surface water is merely water on the earth's surface. The same drop of water moves from above the surface to below the surface and vice versa. Whether above or below, it follows the same physical laws at all times. Consideration of this fact should remove a part of the mystery which sometimes seems to surround the subject of ground water. The facts concerning movement of water underground are more difficult to obtain than those for water on the surface and this has caused less definite application of well-known principles than has been the case with water on the surface, but accumulating knowledge is giving methods for, and sureness in, dealing with ground water. This paper has presented, briefly, some of the values found and methods so far achieved, keeping in mind always that a solution of any water problem of the nature discussed in this paper is accomplished by merely substituting correct values in a simple equation. It is

the values which are often difficult to determine correctly; but as data accumulate this difficulty decreases.

Ground water in alluvium is moving as a stream but at the same time the analytical approach to determination of yield is often, probably generally, best made if the ground-water body is treated as a reservoir. Regardless of this a clearer view will be had if the conception of it as a stream is not lost.

Each ground-water problem is unique in some of its elements. Even though the field has been very narrowly delimited, it is still so broad that it cannot be handled with anything approaching adequacy in a paper such as this; but, if the paper has served in any way to outline the possibilities in ground-water utilization and will promote discussion of the general engineering approach to such utilization, it has served its purpose.

APPENDIX

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

INTERRELATION OF CERTAIN STRUCTURAL CONCEPTS

BY CAMILLO WEISS,[1] M. AM. SOC. C. E.

SYNOPSIS

A study in restraint as it affects engineering structures is presented in this paper. Physical interpretation is given to such structural terms as "fixed point," "carry-over factor," and "degree of restraint," and their mutual relations are established.

INTRODUCTION

The analysis of stresses and bending moments in any indeterminate structure is dependent upon the sections of the members comprising this structure. Because of the rapid increase in the number and variety of structures involving indeterminate frames there has been a correspondingly rapid increase in the methods for the computation of the stresses and moments in such frameworks.

The method of moment distribution introduced by Hardy Cross,[2] M. Am. Soc. C. E., offers a relatively rapid and precise means of computation and, in the writer's opinion, is as good a method as can be devised. It permits the analysis of a framework from a unique point of view. Whereas the method of least work is based on the concepts of work and energy; the slope-deflection method on relations governing deformations comprising the geometry of flexure; the Cross method may be considered as based on a complex of concepts that can be summed up under the term "restraint."

Work and deformation imply the action of a force. Restraint, however, is entirely a function of the structure itself. In this study an attempt has been made to correlate some of the component parts that characterize restraint. It is a purely academic study and no new methods are proposed.

The structural concepts primarily considered are the "fixed point," the "carry-over factor," and the "degree of restraint." "Fixed points" have been defined as points of zero bending moment, or fixed points of contraflexure, for

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by June 1, 1945.

[1] Designer, Bethlehem Steel Co., Fabricated Steel Constr., Eng. Dept., Bethlehem, Pa.

[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

unloaded spans.[3] Usually such points are thought of in connection with beams continuous over three or more supports; but two such "fixed points" exist in every member of any frame. They are called "fixed" because their position on the elastic curve is stationary. In this study the shorter term "node" has been used to designate these characteristic points.

For the purpose of distributing fixed-end moments, Professor Cross defined the carry-over factor as the ratio of the moment induced at the fixed end to the moment applied at the rotating end of a member which is on unyielding supports at both ends. In this study the term "carry-over factor" denotes the ratio of the moment induced at the restrained, but not necessarily fixed, end to the moment applied at the other end of a member which is on unyielding supports at both ends.

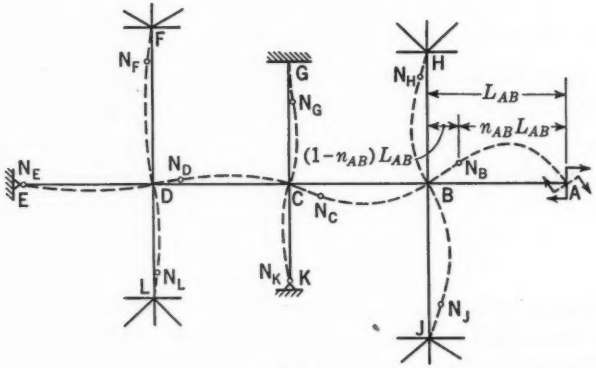


FIG. 1

"Degree of restraint" is a term which has been used loosely. A general definition of the concept is offered in this study and a law is established which states that, for m members rigidly connected to form a joint, the sum of their degrees of restraint equals $(m - 1)$. A "degree of fixation" has been defined by Thomas F. Hickerson,[4] M. Am. Soc. C. E. It is a useful concept and is a special case of this general degree of restraint. The interrelation of these three concepts is derived analytically and, to a limited extent, illustrated graphically.

NOTATION

The letter symbols used in this paper are defined where they first appear, in text or illustration, and are assembled for convenience of reference in Appendix I.

FIXED POINTS AND CARRY-OVER FACTORS

When one end of a restrained member is rotated back and forth, the wave-like elastic curves will pass through a certain node. In the special case of the beam continuous over several supports, this node is one of the "fixed

[3] "Continuous Bridges," by D. B. Steinman in "Movable and Long-Span Bridges," by G. A. Hool and W. S. Kinne, 1923, pp. 250–252.

[4] "Statically Indeterminate Frameworks," by Thomas F. Hickerson, Univ. of North Carolina Press, Chapel Hill, 1937, pp. 8–10.

points" of the given span. A similar node will occur in a member AB of any framework when one end A is rotated as illustrated in Fig. 1. Its location on the axis of the member is stationary and is a function of the stiffness of the member and of the restraint at the far end. The node (points N, Fig. 1) caused by end rotation is at the far end of the member when that end is not restrained, or at some point between the far end and the third point nearest that end, this latter position being the position for full fixity at that end. Hence the nodal intercept measured from the generating end (end A in member AB; end B in members BC, BH, and BJ, etc.) is generally more than twice the nodal intercept measured from the restrained end.

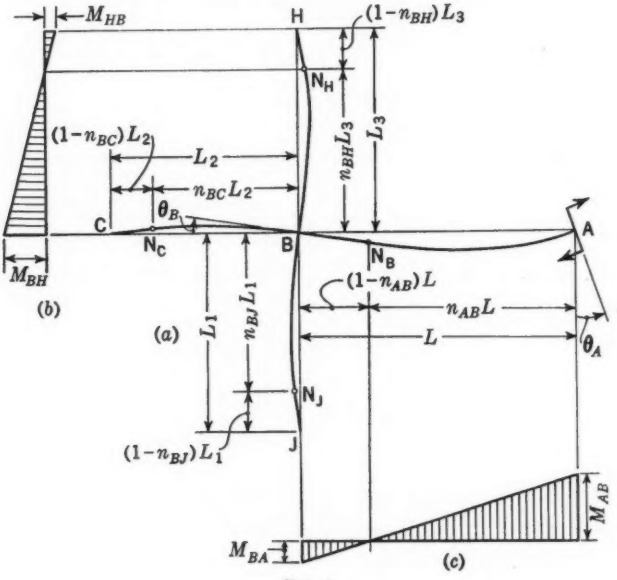


FIG. 2

The node divides each member in the ratio (n) which the generating moment bears to the induced moment. This property of the node furnishes a physical concept for a general carry-over factor f; that is, the carry-over factor is the ratio of the shorter nodal intercept to the longer nodal intercept.

From end A to end B the carry-over factor is

$$f_{AB} = \frac{1 - n_{AB}}{n_{AB}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

When the induced moment occurs at a fixed end, $n_{AB} = \frac{2}{3}$ and the ratio is $\frac{1}{2}$, which is defined as the carry-over factor for beams of constant cross section by Professor Cross.[1]

NODAL INTERCEPT RATIOS

The unloaded beam AB, Fig. 2(a), is restrained at point B by unloaded members BJ, BC, and BH. A moment applied at end A causes the joint to

rotate through an angle θ_A. Elastic distortion of member AB induces rotations θ_B, θ_J, θ_C, and θ_H of joints B, J, C, and H, respectively, and distortion of the members meeting at those points. Assuming that no relative displacement occurs—

$$M_{BH} = 2\,E\,K_{BH}\,(2\,\theta_B + \theta_H) \quad \ldots\ldots (2a)$$

and

$$M_{HB} = 2\,E\,K_{BH}\,(2\,\theta_H + \theta_B) \quad \ldots\ldots (2b)$$

Dividing Eq. 2*b* by Eq. 2*a* gives the relation

$$\frac{M_{HB}}{M_{BH}} = \frac{2\,\theta_H + \theta_B}{2\,\theta_B + \theta_H} \quad \ldots\ldots (3a)$$

Referring to Fig. 2(*b*) it follows from the similarity of triangles that

$$\frac{M_{HB}}{M_{BH}} = \frac{1 - n_{BH}}{n_{BH}} \quad \ldots\ldots (3b)$$

By combining Eqs. 3*a* and 3*b* it is possible to express θ_H in terms of θ_B:

$$\theta_H = -\,\theta_B \frac{3\,n_{BH} - 2}{3\,n_{BH} - 1} \quad \ldots\ldots (4)$$

Likewise, the slopes of the other joints adjacent to joint B can be expressed in terms of θ_B:

$$\theta_J = -\,\theta_B \frac{3\,n_{BJ} - 2}{3\,n_{BJ} - 1} \quad \ldots\ldots (5a)$$

$$\theta_C = -\,\theta_B \frac{3\,n_{BC} - 2}{3\,n_{BC} - 1} \quad \ldots\ldots (5b)$$

and

$$\theta_H = -\,\theta_B \frac{3\,n_{BH} - 2}{3\,n_{BH} - 1} \quad \ldots\ldots (5c)$$

Similarly (compare Eqs. 3 and 4):

$$\frac{M_{BA}}{M_{AB}} = \frac{2\,\theta_B + \theta_A}{2\,\theta_A + \theta_B} = \frac{1 - n_{AB}}{n_{AB}} \quad \ldots\ldots (6a)$$

and

$$\theta_A = -\,\theta_B \frac{3\,n_{AB} - 1}{3\,n_{AB} - 2} \quad \ldots\ldots (6b)$$

The condition for static equilibrium of joint B requires that

$$M_{BJ} + M_{BC} + M_{BH} + M_{BA} = 0 \quad \ldots\ldots (7)$$

Expressing the moments in terms of slopes similar to Eqs. 2 and utilizing Eqs. 4 and 6*b*,

$$2\,E\,K_{BJ}\frac{3\,n_{BJ}}{3\,n_{BJ} - 1} + 2\,E\,K_{BC}\frac{3\,n_{BC}}{3\,n_{BC} - 1} + 2\,E\,K_{BH}\frac{3\,n_{BH}}{3\,n_{BH} - 1} + 2\,E\,K_{AB}\frac{3\,n_{AB} - 3}{3\,n_{AB} - 2} = 0 \quad \ldots\ldots (8)$$

Dividing by 6 E and rearranging Eq. 8,

$$\frac{1 - n_{AB}}{3\,n_{AB} - 2} = \frac{K_{BJ}}{K_{AB}} \times \frac{n_{BJ}}{3\,n_{BJ} - 1} + \frac{K_{BC}}{K_{AB}} \times \frac{n_{BC}}{3\,n_{BC} - 1} + \frac{K_{BH}}{K_{AB}} \times \frac{n_{BH}}{3\,n_{BH} - 1} + \cdots \quad (9)$$

Eq. 9 may be considered a general equation applying to any joint of the type shown in Fig. 3. It gives the location of the node which characterizes the restraint at end B of a member AB with constant moment of inertia if the remote nodes of the restraining members are known.

Stiffness Factors and Distribution Factors

For a member with a constant ratio $K = \frac{I}{L}$, the term $4\,E\,K$ is a measure of its maximum stiffness. It expresses the resistance to rotation of one end of a member if the other end is rigidly fixed. If the restraint at the far end is relaxed until it is free to turn, the resistance to rotation of the near end will diminish from $4\,E\,K$ to $3\,E\,K$.

In general the resistance to rotation of the end of a member will be E times a stiffness factor or restraint factor "r." For a member with constant moment of inertia, r then varies between $3\,K$ and $4\,K$.

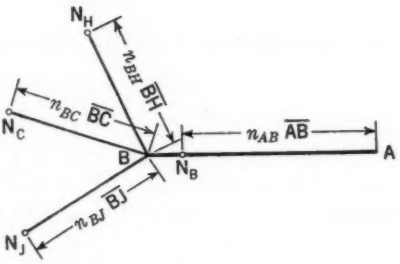


Fig. 3

Referring again to Eqs. 7 and 8 and to Figs. 2 and 3, it is evident that, provided lateral displacement of joints is prevented, each member BX which restrains end B of member AB contributes to the restraint a quantity E times r, in which the stiffness factor or restraint factor r equals

$$r_{BX} = 6\,K_{BX}\,\frac{n_{BX}}{3\,n_{BX} - 1} \quad (10)$$

Assuming that all members are made of the same material, E is a constant. It will be omitted hereafter but should not be forgotten. Relatively, then, each member contributes to the restraint of end B of member AB a quantity:

$$r_{BJ} = \frac{6\,n_{BJ}}{3\,n_{BJ} - 1}\,K_{BJ} \quad (11a)$$

$$r_{BC} = \frac{6\,n_{BC}}{3\,n_{BC} - 1}\,K_{BC} \quad (11b)$$

and

$$r_{BH} = \frac{6\,n_{BH}}{3\,n_{BH} - 1}\,K_{BH} \quad (11c)$$

The total restraint R_{BA} exerted upon end B of member AB equals the sum of the restraint factors:

$$R_{BA} = r_{BJ} + r_{BC} + r_{BH} \quad (12)$$

The quantity

$$d_{BA} = \frac{r_{BA}}{R_{BA} + r_{BA}} \quad \ldots\ldots (13)$$

measures the ratio in which an external moment applied at the joint B will be distributed, and has been called a "distribution factor."

Eqs. 1, 9, and 10 give the carry-over factor, the nodal intercept ratio, and the stiffness factor or restraint factor in terms of the nodal intercept ratio. Nodal intercept ratio, restraint factor, and carry-over factor can be expressed also in terms of the restraint, R, as follows: By combining Eqs. 9, 10, and 12 and solving for n_{AB}, the nodal intercept ratio is equal to

$$n_{AB} = \frac{2}{3}\frac{R_{BA} + 3\,K_{AB}}{R_{BA} + 2\,K_{AB}} \quad \ldots\ldots (14)$$

By combining Eqs. 10 and 14, the stiffness factor or restraint factor is equal to

$$r_{AB} = 4\,K_{AB}\frac{R_{BA} + 3\,K_{AB}}{R_{BA} + 4\,K_{AB}} \quad \ldots\ldots (15)$$

By combining Eqs. 1 and 14, the carry-over factor is equal to

$$f_{AB} = \frac{1}{2}\frac{R_{BA}}{R_{BA} + 3\,K_{AB}} \quad \ldots\ldots (16)$$

If end B is hinged, $R_{BA} = 0$ and Eqs. 14 to 16 furnish $n_{AB} = 1$; $r_{AB} = 3\,K_{AB}$; and $f_{AB} = 0$. If end B is fixed, $R_{BA} = \infty$, and Eqs. 14 to 16 furnish $n_{AB} = \frac{2}{3}$; $r_{AB} = 4\,K_{AB}$; and $f_{AB} = \frac{1}{2}$.

Degree of Restraint

The term "degree of restraint" is generally applied very loosely. Based on the preceding discussion the degree of restraint Q_{AB} will be defined as the ratio of the restraint exerted upon the end of a member to the resistance against rotation of the entire joint:

$$Q_{AB} = \frac{R_{AB}}{R_{AB} + r_{AB}} \quad \ldots\ldots (17a)$$

The degree of restraint can be defined also as the difference between unity and the distribution factor, because

$$Q_{AB} = \frac{R_{AB}}{R_{AB} + r_{AB}} = 1 - \frac{r_{AB}}{R_{AB} + r_{AB}} \quad \ldots\ldots (17b)$$

It can be shown that for any rigid joint of m members the summation of their degrees of restraint as defined by Eqs. 17 must equal $(m - 1)$; that is:

$$\sum Q = m - 1 \quad \ldots\ldots (18)$$

For instance, if four members form a rigid joint, the summation $\sum Q = 4 - 1 = 3$, and the average restraint equals 75%. If any of the four members have a degree of restraint greater than 75%, the restraint on one or more of

the other three members is correspondingly reduced, and vice versa. If two members form a joint and one member restrains the other 100% (as, for instance, a member rigidly embedded in the earth), the second member exerts no restraint on the first member.

In terms of nodal intercept ratios Eq. 17*a* may be written

$$Q_{AB} = \frac{(1 - n_{BA})(3\,n_{AB} - 1)}{(1 - n_{BA})(3\,n_{AB} - 1) + n_{AB}(3\,n_{BA} - 2)} \quad \ldots\ldots\ldots (19)$$

Obviously, as both n_{AB} and n_{BA} are contained in Eq. 19, the degree of restraint at one end is a function also of the degree of restraint at the other end of a member.

Degree of Fixity

"Degree of fixity" (or "degree of fixation") is another term commonly used in the study of indeterminate structures. It has been defined[4] in terms of end slopes as follows:

> Applying a rotation θ_B to end B of an unloaded member AB, the degree of fixity at end A is the difference between unity and the ratio of the slope at the restrained end A to the slope at A if the end were hinged.

From this definition the expressions for the carry-over factor, stiffness factor, nodal intercept ratio, and degree of restraint in terms of the degree of fixity are as follows:

$$f_{AB} = \frac{2\,F_{BA}}{3 + f_{BA}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

$$r_{AB} = K_{AB}(3 + F_{BA}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

$$n_{AB} = \frac{1}{3}\frac{3 + F_{BA}}{1 + F_{BA}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20c)$$

and

$$Q_{AB} = \frac{4\,F_{AB}}{4\,F_{AB} + (1 - F_{AB})(3 + F_{BA})} \quad \ldots\ldots\ldots\ldots\ldots (20d)$$

Some Mutual Relations

By definition the degree of fixity F_{BA} may vary between zero and unity. From Eq. 20*d* it can be seen that the degree of restraint, Q_{AB}, may vary, therefore, between $\frac{4\,F_{AB}}{3 + F_{AB}}$ and F_{AB}. In other words, $Q'_{AB} = F_{AB}$ when $F_{BA} = 1$. In terms of the nodal intercept ratio this means that $Q'_{AB} = F_{AB}$ when $n_{AB} = \frac{2}{3}$ (see Eq. 20*c*) and, from Eq. 19, it follows that

$$Q'_{AB} = F_{AB} = \frac{3 - 3\,n_{BA}}{3\,n_{BA} - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

It is further interesting to note that, if end B is hinged and $n_{AB} = 1$, Eq. 19 reduces to

$$Q''_{BA} = \frac{2(1 - n_{BA})}{n_{BA}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22a)$$

Referring to Eq. 1 it is seen that

$$Q''_{AB} = 2 f_{BA} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22b)$$

This can be demonstrated also as follows: Let C_{AB} be the fixed-end moment at end A due to an external moment M_B at end B. Then $C_{AB} = \frac{1}{2} M_B$. The end moment at end A is $M_{AB} = Q_{AB} C_{AB}$ and by definition

$$f_{BA} = \frac{M_{AB}}{M_B} = \frac{Q_{AB} C_{AB}}{2 C_{AB}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

from which $Q_{AB} = 2 f_{BA}$.

Eqs. 21 and 22 have been plotted in the chart in Table 1, which shows graphically the restraint characteristics of the joint. If for a given member AB the nodal intercept ratio n_{BA} is determined, which is a function of the characteristics of joint A only, the following information is known:

(1) The degree of fixity at joint A, as this lies on the lower curve;

(2) The carry-over factor from joint B to A, which is one half the value of the ordinate of the upper curve; and

(3) The fact that the degree of restraint (and, therefore, the distribution factor) for end A of a member AB for any external moment at joint A is the ordinate of a point A somewhere between the two curves depending on the restraint at end B. This very narrow range demonstrates the well-known fact that (especially with increasing remoteness) adjacent restraints do not affect, materially, the distribution of an external moment at a joint.

Moment Flow

An external moment (such as an unbalanced fixed-end moment) is held in equilibrium by all the members forming the joint at which it is acting and is distributed among all of these members in the ratio of their distribution factors. A moment that has been carried over to a joint by any one member is held in equilibrium by the other members of that joint and is distributed to these other members.

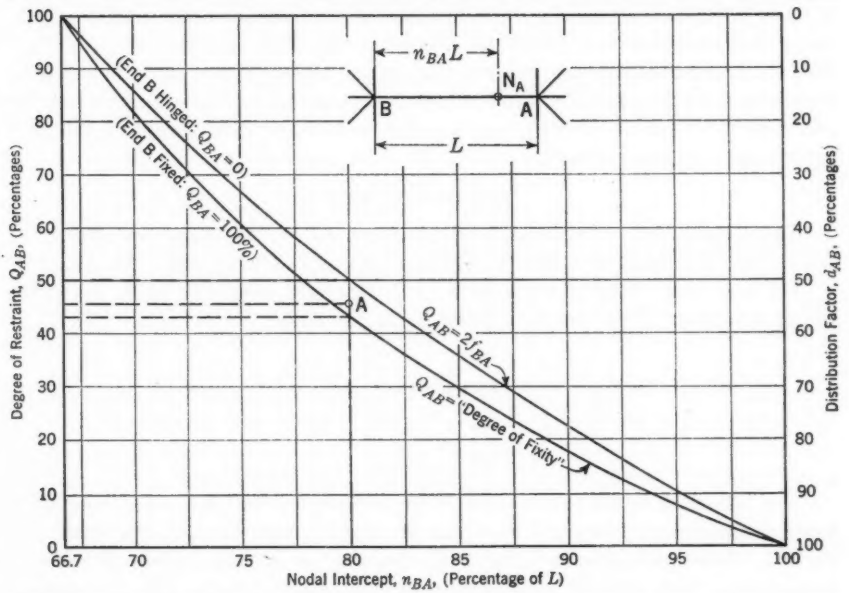
An unbalanced fixed-end moment or an external moment thus flows from joint to joint, diminishing as it spreads and leaving each joint in equilibrium. In a closed frame the result is an infinite series of waves. It is well known that convergence is so rapid, however, that for all practical purposes it is permissible to insolate a part of a frame by assigning arbitrary restraints to the marginal joints. In an open frame the necessary constants can generally be determined and the moments obtained by one distribution.

Some of the relations established in this paper are illustrated in Table 1. Attention is called to the fact that all functions are based on constant moments of inertia and on absence of joint displacements.

Summary

In an attempt to generalize and correlate certain concepts used in the analysis of structures, and assuming constant cross sections, simple algebraic relations were found to exist between some of these terms which in effect reduce them to a "common denominator."

TABLE 1.—INTERRELATION BETWEEN THE NODE, DEGREE OF RESTRAINT, DEGREE OF FIXITY, AND CARRY-OVER FACTOR



Structural concept	Symbol	Structural Concept in Terms of: Degree of fixity	Restraint factors	Nodal intercept ratio
Nodal intercept ratio	n_{AB}	$\frac{1}{3}\frac{3+F_{BA}}{1+F_{BA}}$	$\frac{2}{3}\frac{R_{BA}+3K_{AB}}{R_{BA}+2K_{AB}}$	(See Eq. 9 and Fig. 3)
Distribution factor	d_{AB}	$\frac{(1-F_{AB})(3+F_{BA})}{(1-F_{AB})(3+F_{BA})+4F_{AB}}$	$\frac{r_{AB}}{R_{AB}+r_{AB}}$	$\frac{n_{AB}(3n_{BA}-2)}{n_{AB}(3n_{BA}-2)+(1-n_{BA})(3n_{AB}-1)}$
Carry-over factor	f_{AB}	$\frac{2F_{BA}}{3+F_{BA}}$	$\frac{1}{2}\frac{R_{BA}}{R_{BA}+3K_{AB}}$	$\frac{1-n_{AB}}{n_{AB}}$
Degree of restraint	Q_{AB}	$\frac{4F_{AB}}{4F_{AB}+(1-F_{AB})(3+F_{BA})}$	$\frac{R_{AB}}{R_{AB}+r_{AB}}$	$\frac{(1-n_{BA})(3n_{AB}-1)}{(1-n_{BA})(3n_{AB}-1)+n_{AB}(3n_{BA}-2)}$
Degree of fixity	F_{AB}	....	$\frac{R_{AB}}{R_{AB}+4K_{AB}}$	$\frac{3-3n_{BA}}{3n_{BA}-1}$
Restraint factor (or stiffness factor)	r_{AB}	$K_{AB}(3+F_{BA})$	$4K_{AB}\frac{R_{BA}+3K_{AB}}{R_{BA}+4K_{AB}}$	$K_{AB}\frac{6n_{AB}}{3n_{AB}-1}$

The approach to the subject matter being essentially mathematical, a problem incidental thereto, but important none the less, presents itself. It is the question of adopting some form of uniform mathematical language—algebraic letter symbols—for the expression of commonly recognizable terms. In Appendix I, the writer has attempted to conform essentially with standard lists now in process of formulation. It is acknowledged that the effort is

imperfect, and that there is need for objective thought in the compilation of such a "dictionary."

In the hope that some readers may have suggestions to offer in discussion, a list of Tentative American Standard Letter Symbols for Structural Analysis, being perfected currently under the chairmanship of Albert Haertlein, M. Am. Soc. C. E., is added to the paper as Appendix II. It is not intended to open for discussion the entire field of structural symbols, but any suggestions for adapting the symbols of the present paper to the proposed standard will be a contribution to this subject.

Acknowledgment

This study was submitted in draft form to W. H. Jameson and J. L. Walmsley, Members, Am. Soc. C. E., and George L. Gray, Assoc. M. Am. Soc. C. E., and their helpful comments are gratefully acknowledged, as is the assistance given by Sidney Liu in preparing the copy.

APPENDIX I

Notation

The following letter symbols, used in this paper, conform essentially with American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials[5] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

Subscripts.—Capital letter subscripts are used to specify a given location on the frame. For example, M_{AB} denotes the moment at end A of a beam AB; or K_{AB} denotes the stiffness ratio I/L for the beam AB.

d = distribution factor for an unbalanced moment;
E = modulus of elasticity in tension and compression;
F = degree of fixity at the points indicated by subscripts;
f = carry-over factor = the ratio of the shorter nodal intercept to the longer nodal intercept (Eq. 1);
I = moment of inertia;
K = a constant ratio = I/L, or stiffness ratio;
L = length; total span length between joint intersections;
M = external moment;
n = ratio of the longer of two nodal intercept distances, to the length L; for brevity this ratio is called "the nodal intercept ratio";
Q = degree of restraint (see Eq. 17*a*);
R = summation of restraint or stiffness factors r, of all beam ends exerting a restraint on the end of a given beam (see Eq. 12);
r = restraint factor or stiffness factor at the end of a given beam (see Eq. 10);
θ = change in the slope of the tangent to the elastic curve at the end of a given beam.

[5] ASA—Z10a—1932.

APPENDIX II

Proposed American Standard Letter Symbols for Structural Analysis

In the following the general principles of Letter Symbol Standardization were formulated by a Committee of the American Standards Association, with Society representation, and have been adopted by the Association as a guide for the selection of letter symbols in all fields. The list of structural symbols is a tentative one, approaching final action by the Association.

General Principles of Standardization

1. Manuscript.—In preparing manuscripts, it is suggested that authors give careful attention to the use of symbols from this and other standard lists and to the principles here given. Symbols used should be defined clearly. When a table of symbols is not given, it is desirable to make reference to the standard lists from which the symbols are taken. The many numbers, letters, and signs which are similar in appearance should be distinguished carefully.

2. Letter Symbol.—A letter symbol is a single character, with subscript or superscript if required, used to designate a physical magnitude in mathematical equations and expressions. Two or more symbols together always represent a product.

Letter symbols are to be distinguished from abbreviations, mathematical signs and operators, graphical symbols, and chemical symbols:

(*a*) Abbreviations are shortened forms of names and expressions employed in texts and tabulations and should not be used as symbols in equations.

(*b*) Mathematical signs and operators are characters used with letter symbols to denote mathematical operations and relations.

(*c*) Graphical symbols are conventionalized diagrams and letters used on plans and drawings.

(*d*) Chemical symbols are letters and other characters designating chemical elements and groups.

3.—The same symbol should be used for the same physical magnitude regardless of the units employed and regardless of special values occurring for different states, points, parts, times, etc. Units or special values may be distinguished when necessary by subscripts, superscripts, or by upper-case and lower-case letters when both are specifically included as symbols in a standard list. The units used should be indicated when necessary. Sometimes different symbols are used for the components of a vector.

4. Subscript.—A subscript preferably should be a simple character. It is commonly employed to indicate constancy of a particular physical magnitude, such as pressure or temperature, when there are other variables. A multiple subscript, sometimes divided by a comma, refers to more than one state, point, part, time, etc. A subscript should not be attached to a subscript. Further uses of subscripts are listed in Principles 3 and 6.

5. Superscript.—A symbol with a superscript, such as a prime (′) or a second (″), should be enclosed in parentheses, braces, or brackets before affixing an exponent. A complicated exponent (or any other expression frequently repeated) may be replaced by a single symbol selected to represent it. Reference marks should not be attached to symbols. Further uses of superscripts are listed in Principles 3 and 6.

6. Conflicts.—Conflicts which would occur when different physical magnitudes are assigned the identical symbol in the same or different standard symbol lists may be resolved in one of the following ways:

(*a*) For one or more of the conflicting uses, the given symbol may be employed with subscript or superscript selected by the author.

(*b*) If one of the magnitudes has an alternate symbol in a standard list, it may be used.

(*c*) A slight change in the name of the magnitude may remove the conflict. For instance, one may use L for "length of radius" when r for "radius" conflicts with r used for another magnitude.

7. Unlisted Magnitudes.—To symbolize a special value of a listed magnitude, see Principle 3. The symbol chosen by an author for a physical magnitude not appearing in any standard list should be one that does not have already a different meaning in the field of the text.

8. Typography.—Letter symbols, letter subscripts, and letter superscripts, whether upper case or lower case, should be printed with italic type unless definitely specified otherwise. On manuscript this is indicated by underlining each symbol which is to be italicized. Special types, such as Old English and type that is currently used for vector magnitudes, should be avoided for scalar magnitudes. When special type is used for vector magnitudes, the same italic letter should be used for the corresponding scalar magnitude. Vertical Arabic numerals should be used as coefficients in equations and in subscripts and exponents. Bars, dots, and other modifying signs and operators should be used in the manner currently recognized by mathematicians.

Tentative List of Symbols

General Letter Symbols.—

A = area;
b = breadth or width;
D = diameter;
E = modulus of elasticity (Young's modulus);
e = eccentricity;
F = force (see also P);
f = coefficient of sliding friction;
G = modulus of elasticity in shear, or rigidity;
g = acceleration of gravity;
h = height, depth, or thickness;
I = inertia, rectangular moment of (area); I_{xy} = product moment of inertia;

J = polar moment of inertia;
k = radius of gyration;
L = length;
M = moment of force, bending moment;
P = forces or loads, concentrated (see also F);
p = pressure, normal force per unit area;
Q = moment of area (statical, or first moment);
R = modulus of rupture;
r = radius;
s = normal stress per unit area;
s_s = shear stress per unit area;
T = torque;
t = temperature;
V = volume;
W = weight, total;
w = load, or weight per unit distance;
x = coordinate distance measured parallel to an x-axis;
y = coordinate distance measured parallel to a y-axis; deflection, or ordinate to the elastic line of a beam at distance x;
Z = section modulus;
α = coefficient of thermal expansion, linear;
γ = weight per unit volume;
δ = total elongation;
ϵ = unit linear deformation; normal strain;
η = efficiency;
μ = Poisson's ratio;
ρ = density; mass per unit volume, $\frac{\gamma}{g}$;
ϕ = total angular deformation.

Letter Symbols Primarily for Reinforced Concrete.—

C = total compressive force;
c = distance from the neutral axis to the extreme fiber;
d = depth from the compression face of a beam or slab to the center of the longitudinal tensile reinforcement;
f = unit stress; f_c = compressive stress in concrete and f_s = tensile stress in steel reinforcement;
j = ratio of the distance between the resultant compressive stress and the resultant tensile stress to the distance from the outer compressive fiber to the resultant tensile stress in a reinforced concrete beam;
n = ratio, modulus of elasticity of steel to modulus of elasticity of concrete;
p = ratio, area of steel reinforcement to area of concrete;
T = total tensile force on the steel reinforcement;
u = unit bond stress;
v = unit shearing stress.

Letter Symbols Primarily for the Analysis of Frameworks, Arches, and Cables.—

c = distance from the neutral axis to the extreme fiber;
h = rise of an arch or sag of a cable;
K = stiffness factor = $\frac{I}{\text{span}}$;
M = moment of a force; bending moment;
m = moment at any section of a beam due to a unit load or to a unit moment applied at a given point;
R = reaction;
s = linear distance, or length measured along an arc;
u = force at any bar of a framed structure due to a unit load applied at a given point in a given direction;
V = shearing force in a beam section;
y = deformation; total displacement of a panel point of a truss or a point on the elastic line of a beam;
Z = section modulus;
γ = shear strain;
δ = total elongation;
ϵ = unit linear deformation; normal strain;
θ = angular distance in slope deflection;
ϕ = total angular deformation.

Letter Symbols Primarily for Mechanics of Solid Bodies.[6]—

a = linear acceleration;
E = energy, E_k being kinetic energy and E_p being potential energy (use T and V, respectively, when there is a conflict with E = modulus of elasticity);
f = frequency;
K = stress concentration factor;
T = kinetic energy (E_k is preferred except when there is a conflict with E = modulus of elasticity);
t = time;
V = potential energy (E_p is preferred except when there is a conflict with E = modulus of elasticity);
v = linear velocity;
W = work;
α = angular acceleration;
ω = angular velocity.

[6] See also American Standards Association Letter Symbols for Mechanics of Solid Bodies, ASA—Z10.3—1942.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DEWATERING, INCINERATION, AND USE OF SEWAGE SLUDGE

A SYMPOSIUM

Discussion

BY C. R. VELZY, AND GEORGE J. SCHROEPFER

C. R. VELZY,[13] M. AM. SOC. C. E.[13a]—The Symposium and the discussions describe the problems of dewatering and incineration, show where improvements and economies can be achieved, and indicate the possible direction of future developments. In general, conclusions are not drawn but rather experiences are stated with the inference that study and development should be continued.

The disposal of sludge thus far has been a necessary expense of sewage treatment. Disposal processes do not affect the quality of treatment plant effluent, and the value of sludge has never been more than that of a by-product. In some cases, utilization of sludge for fertilizer has yielded an income, and this use will develop as costs of dewatering and handling are reduced and as the demand increases for sludge as a soil conditioner. Efforts toward improvements in sludge disposal in existing plants, therefore, have had, as a final objective, reduction in cost of the processes.

Attention is called to the need for a high degree of ability in the directive personnel, and skill and interest on the part of operators, of dewatering and incineration plants. This need is obvious, not only on account of the complexity of the processes, but also in order to carry on the research and development necessary to continued progress.

The description of the problems of dewatering and incineration, and the comments on utilization of sludge are instructive but do not point to any common procedure for sludge disposal which may be followed in all cases. Designing engineers must still make careful analyses for each new design. For

NOTE.—This Symposium was published in January, 1943, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1943, by A. L. Genter; June, 1943, by Willem Rudolfs; September, 1943, by Messrs. Lloyd M. Johnson, and Isador W. Mendelsohn; and February, 1944, by Langdon Pearse.

[13] Asst. Engr., Greeley & Hansen, New York, N. Y.

[13a] Received by the Secretary February 21, 1944.

small plants, the use of sand-drying beds is most economical, with utilization of sludge for fertilizer dependent largely on local demand. In some sea coast cities, disposal of sludge at sea is still the least costly procedure. For the large group of cities not in either of these classifications, a thorough analysis of each case is required. Location of the plant, availability of space, and other factors will affect the selection of sludge disposal processes, but the over-all cost of sludge disposal is likely to be the controlling factor.

Experience in existing plants indicates that design should always provide adaptability for future developments. One important item in this connection is the possibility of the use of sludge for fertilizer. The economy of this development may depend upon cost of auxiliary fuel and the market for fertilizer. An incinerating plant which has flexibility in operation as between incineration and drying of sludge for fertilizer has an advantage in being able to follow market conditions. Other possible developments, such as better temperature control in incinerators, heat recovery, and improvements in mechanical equipment, should be considered in design so that observations may be made and adjustments effected, as indicated by the operating experience.

GEORGE J. SCHROEPFER,[14] ASSOC. M. AM. SOC. C. E.[14a]—Many of the problems arising in connection with various types of sludge disposal are presented in the papers of this Symposium and in the discussions. Some solutions have been suggested. Years ago an eminent sanitary engineer made a statement to the effect that "Sewage treatment is sludge disposal," indicating that the controlling problem is the disposal of sludge resulting from various processes of sewage treatment. A reading of these papers renews the conviction of the writer that this statement is still apparently true. Frank discussion of some of these problems, as in this Symposium, is leading to their satisfactory solution, and toward even further improvement in the art of sewage treatment.

The writer enjoyed reviewing the various discussions, and was particularly pleased by Mr. Genter's thorough consideration of the results of the Minneapolis-St. Paul operations. Mr. Genter has accumulated a mass of valuable information on sludge disposal which he presents very effectively, from his point of view, in Table 19. He uses the results of this analysis as the basis of a major part of his discussion. Unfortunately, Mr. Genter bases his comparisons on per capita values which the writer will attempt to show are not scientific and in accordance with the facts. For this reason, the writer's review of Mr. Genter's discussion is limited to showing the fallacy of the method of comparison used.

Every one conversant with the field of sewage treatment realizes that there are variations, affected by a number of factors, in the quantity and quality of sewage from various municipalities. Furthermore, and in addition, there is a large variation in the removals accomplished by different plants. A comparison of costs and efficiencies of sludge disposal on a per capita basis penalizes a plant with a treatment process that is more effective than another or which by chance happens to receive a relatively stronger sewage. Since the per capita

[14] Chf. Engr. and Supt., Minneapolis-St. Paul San. Dist., St. Paul, Minn.

[14a] Received by the Secretary December 23, 1943.

quantities of solids removed may vary as much as tenfold between two plants, the per capita basis of comparing sludge disposal is improper and unsound. Although this method of comparison may have a place in commercial literature, or in general comparisons for popular consumption, its usefulness or propriety in scientific analysis is open to serious question.

In Table 21 some of Mr. Genter's data from Table 19 are included, and, in addition, but using the basic data, other comparisons are shown (with items such as Nos. 2a, 26a, 26b, and 28a). Considering the various data (see item No. 2a), the quantity of sewage in gallons per capita per day varies for the plants listed from 77 to 234. The suspended solids content (item No. 3) varies from 155 to 329 ppm. The percentage removal of solids by the plants listed varies from 34.8% to 74.6%. All of these factors combine to cause a large variation in the quantity of solids removed and to require disposal in the various plants, ranging from 33.4 to 89.5 lb per capita per yr. In the same manner the quantity of sludge filtered varies from 12.3 to 89.5 lb per capita per yr. By the writer's method of comparison the latter six plants in Table 21, using elutriation, are shown to be particularly efficacious. The writer insists that to compare, on a per capita basis, six plants employing elutriation, and filtering an average of about 15 lb per capita per yr, with the Buffalo plant (which filters 33.5 lb per capita per yr) and with the Minneapolis-St. Paul plant (which filters 89.5 lb per capita per yr) is unscientific and fallacious.

On a per capita basis (item No. 24, Table 21), the costs at Minneapolis-St. Paul are the highest, \$9.64 per capita per yr, compared to an average of \$5.60 for the six plants employing elutriation. However, on the more proper basis of tonnage of solids removed or disposed of, the order of economy is reversed. Thus, the filtration cost per dry ton of raw solids is \$2.23 at the Minneapolis-St. Paul plant compared to a \$2.60 average for the six plants employing elutriation. Furthermore, even the tonnage of raw solids removed by the settling tanks is not a proper basis for comparing the costs of sludge filtration in several plants in some of which digestion is employed and in others not utilized, since, in a particular plant, costs of such a major item as chemicals vary directly with the quantity of chemicals actually filtered. For this reason, the cost per dry ton of solids actually filtered (item No. 26b) is a more proper method of comparison of filtration costs. At the Minneapolis-St. Paul plant, the cost per ton of solids filtered is \$2.23, compared to an average of \$7.78 in the other six plants (lowest \$4.63, highest \$11.45). Thus, on a more proper basis, the costs at the Minneapolis-St. Paul plant are less than one third of the average costs at the six plants employing elutriation, instead of twice as much as Mr. Genter's basis of comparison implies. The writer concedes that costs at a large plant, such as that at Minneapolis-St. Paul, should be markedly lower than those at a smaller plant filtering as little as one thirtieth of the quantity handled at Minneapolis-St. Paul. The writer also admits that under certain conditions elutriation will result in reduction of chemical requirements for conditioning (possibly at the expense of the treatment process). However, he does not agree that elutriation is the panacea for all sludge disposal "headaches," and in this discussion is using Mr. Genter's own data to show that comparisons on an unsound basis may be very misleading.

TABLE 21.—COMPARISON OF ESSENTIAL ITEMS THAT CONTROL DEWATERING COSTS (1941 DATA)

Item No.	Description	Minneapolis-St. Paul, Minn.	Buffalo, N. Y.	District of Columbia	Hartford, Conn.	Springfield, Mass.	Winnipeg, Manitoba, Canada	San Francisco, Calif.	Annapolis, Md.
	(*a*) BASIC DATA								
1	Approximate population	805,000	600,000	800,000	150,000	130,000	255,000	145,000	25,000
2	Total sewage treated (million gal)	39,286.7	51,262	39,568	5,200	5,799	9,572	2,940.6	703.8
2a	Gallons per capita per day	133	234	135	95	122	106	96	77
3	Suspended solids (ppm)	315	168	180	160	155	329	254	....
4	Percentage removal in plant	74.6	34.8	51	66.5	58.1	54.6	70.5	....
5	Type of sludge dewatered	Raw primary	Digested	Primary digested elutriated	Primary digested elutriated	Primary digested elutriated	Primary digested elutriated	Primary digested elutriated	Primary digested elutriated
	Raw Solids:								
11	Lb per capita per yr	89.5	63	37.5	33.4	49	69	37.9	43.8
	Digested Solids Dewatered:								
13	Lb per capita per yr	....	33.5	17.1	17.3	14.6	13	12.3	13.8
	Percentage of Chemical Dose on Solids:								
17	Ferric chloride	1.53	2.93	3.18	2.23	1.63	2.6	4.3	....
18	Lime (Ca O)	3.77	11.21	....	....	....	....	....	....
19	Ferric sulfate	....	....	....	....	....	....	....	2.86
	(*b*) TOTAL COST OF SLUDGE FILTRATION AND SLUDGE DISPOSAL BY INCINERATION OR HAULING (CENTS PER CAPITA SERVED BY PLANT)								
24	Filtration	9.64	....	3.98	4.25	5.7	5.4	7.0	7.2
25	Incineration or disposal	6.04	....	2.61	1.40	3.22	1.62	1.7	0.0
26	Total	15.68	15.63	6.59	5.65	8.92	7.02	8.7	7.2
26a	Filtration cost per ton of raw solids	2.23	....	2.12	2.63	2.33	1.57	3.70	3.28
26b	Filtration cost per ton of digested solids	2.23	....	4.63	4.92	7.83	8.33	11.45	10.40
	(*c*) CHEMICAL COSTS FOR SLUDGE FILTRATION								
27	Total spent (dollars per yr)	33,410.70	18,382.43	8,500.00	2,060.90	1,108.00	3,803.74	2,300.00	178.70
28	Actual cost, in cents per capita	4.15	3.06	1.06	1.37	0.852	1.49	1.59	0.714
28a	Actual cost per ton of filtered sludge	0.93	1.83	1.24	1.59	1.17	2.30	2.59	1.03

The chemicals required for sludge conditioning vary with the tonnage of sludge filtered. In fact, the chemical requirements are based, as the usual "yardstick" (see Table 19, items Nos. 17, 18, and 19), on the percentage of chemicals as related to the dry sewage solids filtered. Per capita quantities of chemicals have no place in plant comparisons nor in the comparison of year-to-year operation of a particular plant. As a result of the war effort, there has been a marked increase in tonnage of solids received at a particular sewage plant, all out of proportion to the increase in population of the community. Mr. Genter's basis of comparison being on a per capita basis would imply that the engineer or superintendent responsible for the operation of a plant receiving, say, twice the load of solids it formerly received, is operating his plant inefficiently because the quantity of chemicals doubled. Chemical requirements must be based on the tonnage of solids disposed of. Referring to Table 21, and basing the comparison on the costs of chemicals in cents per capita, as Mr. Genter does (item No. 28, Table 19), the costs at Minneapolis-St. Paul are shown to be 4.15¢ as compared to the average of the six plants employing elutriation at 1.18¢. Since these six plants filter only about one sixth of the quantity of solids per capita of the Minneapolis-St. Paul plant, the unscientific nature of this method of comparison will be evident. Basing the comparison on the actual cost of chemicals per ton of solids filtered, the picture is exactly reversed. As compared to a cost per ton of $0.93 at Minneapolis-St. Paul, the average costs for the six other plants shown is $1.64. This difference would possibly represent more the variation in size of the plant with attendant higher unit costs of chemicals than it would the effect of elutriation.

All of the foregoing comparisons have been based on Mr. Genter's data which of necessity were restricted to the period up to 1941. In the case of the Minneapolis-St. Paul plant, further economies in the use of conditioning chemicals have been effected. Thus (see Table 22), as compared to 1941 costs of

TABLE 22.—Annual Comparison of Chemical Requirements and Costs of Sludge Disposal

Year	Percentage of Dry Weight		Actual cost of chemicals per dry ton	Total Operation and Maintenance Cost per Dry Ton		
	Ferric chloride	Lime as Ca O		Vacuum filtration	Incineration	Total
1938	3.17	10.30	2.65	4.92	2.05	6.97
1939	2.10	5.68	1.56	2.64	0.99	3.63
1940	1.83	4.51	1.11	2.19	1.34	3.53
1941	1.53	3.77	0.93	2.16	1.23	3.39
1942	1.20	3.44	0.75	2.10	1.17	3.27
1943[a]	1.10	3.12	0.72	2.05	1.19	3.24

[a] January 1, 1943, to October 31, 1943.

$0.93 per dry ton for chemicals, costs in 1942 were reduced to $0.75, and further lowered for the first ten months of 1943 to $0.72. This is less than one half the costs in the other six plants listed. Similar reductions may have been accomplished in other plants, but these additional data are shown to indicate

that, as a basis of comparison, chemical requirements, even in 1942, had not hit "rock bottom" at this plant.

It is desirable and important that the sludge disposal process employed should not have a deleterious effect on the treatment process. In other words, it is important that solids once removed by the treatment process not be returned as a result of the failure or inability of the sludge disposal process to handle such solids. Within limits, of course, it is reasonable that finely divided solids be returned to the treatment process in the expectation that they will again be removed. In reviewing the data in Table 19, it was observed that in the plants employing elutriation the number of pounds per capita per year removed by sedimentation averaged 45.1, whereas the number of pounds remaining after digestion and elutriation averaged only 14.7 lb. This raises a question of the reason for the disparity between the quantities of sludge actually filtered and the quantity of sludge normally to be expected following digestion but without elutriation. For example, at the Winnipeg plant a total of 69 lb of raw solids per capita per year were removed by sedimentation and discharged into the digestion tanks. However, a total of only 13 lb were filtered, instead of the 35 to 45 lb which might ordinarily be expected with the normal destruction of solids by digestion. Another possible explanation is that some of the solids were disposed of by other means. At one inspection of the Winnipeg plant, this possible explanation was rejected, since no solids were disposed of by any means except filtration. The only remaining possibility, therefore, is that an extremely large part of the solids entering the digestion tanks was "lost" by heavy supernatant pumping from these tanks, or that in the elutriation process certain fine solids were washed out and returned to the raw sewage from which they failed to settle out again. As shown in Table 23, in the six plants employing elutriation following digestion, the average

TABLE 23.—COMPARISON OF EXPECTED AND ACTUAL QUANTITIES OF SLUDGE AT VARIOUS PLANTS, IN POUNDS PER CAPITA PER YEAR

Location	Types filtered[a]	Raw solids	Actually filtered	Expected quantities[b]	Amount Unaccounted For	
					Pounds	Percentage of actual
Minneapolis-St. Paul	R	89.5	89.5	89.5	0	0
Buffalo	D	63.0	33.5	31.5	+2.0	+6.0
Washington	D–E	37.5	17.1	18.7	−1.6	−9.4
Hartford	D–E	33.4	17.3	16.7	+0.6	+3.5
Springfield	D–E	49.0	14.6	24.5	−9.9	−67.8
Winnipeg	D–E	69.0	13.0	34.5	−21.5	−165.0
San Francisco	D–E	37.9	12.3	18.9	−6.6	−53.7
Annapolis	D–E	43.8	13.8	21.9	−8.1	−58.7

[a] Types of sludge filtered: R = raw; D = digested; and D–E = digested and elutriated. [b] Amount that should have been filtered, at 50% reduction in volatile and fixed solids by digestion.

loss of solids was 58.5% of the solids actually filtered, ranging up to 165% in the case of the Winnipeg plant.

All of the foregoing is intended to show that comparisons of sludge disposal should be based on the work done by such processes, that the tonnage basis accurately measures such work, and that a sludge disposal process which ad-

versely affects the treatment process should even be penalized for such effect in a proper basis of comparison.

In the paper by the writer, the advantage of skilled engineering and technical personnel throughout the various grades of a staff in a large plant was emphasized. This observation was commented upon by several of the discussers. When the paper was being written (1941), the Minneapolis-St. Paul plant had been in operation for slightly more than three years, and operating and cost data were included only to October 31, 1941. Since then more data on sludge disposal operations have become available, and these are presented in Tables 22 and 24. In Table 22 it is shown that sludge filtration costs have

TABLE 24.—Comparison of Filtration and Incineration Costs by Years; Minneapolis-Saint Paul Sanitary District (Supplementing Table 10)

Item	Description	1941	1942	1943*
	Filtration Cost, in Dollars:			
1	Operation	70,726.70	64,944.29	....
2	Maintenance	6,862.94	6,184.74	....
3	Total	77,589.64	71,129.03	57,312.51
	Incineration Cost, in Dollars:			
4	Operation	31,943.94	29,517.16	....
5	Maintenance	12,283.49	10,062.18	....
6	Total	44,227.43	39,579.34	33,018.43
7	Dry sewage solids disposed of, in tons	35,993.0	33,838.0	27,740.0
8	Filtration cost, in dollars per dry ton	2.16	2.10	2.05
9	Incineration cost, in dollars per dry ton	1.23	1.17	1.19
10	Total cost of sludge disposal, in dollars per dry ton	3.39	3.27	3.24
11	Cost of entire project (collection, treatment, and sludge disposal), in dollars per dry ton	7.85	8.17	8.25
	Conditioning Chemicals, % of Dry Sewage Solids:			
12	Lime as Ca O	3.77	3.44	3.12
13	Ferric chloride	1.53	1.20	1.10
	Cost of Chemicals, in Dollars per Dry Ton:			
14	Lime at $9.30 per ton (Ca O)	0.35	0.32	0.29
15	Ferric chloride at $39	0.60	0.47	0.43
16	Total	0.95	0.79	0.72
17	Cost of chemicals to filter 110 tons of dry solids, in dollars per day	104.50	86.90	79.20

* To October 31, 1943.

been reduced to $2.05 per dry ton and incineration costs to $1.19. As shown in Table 24, the costs of chemicals, on the basis of 110 tons per day of dry solids and fixed unit costs of chemicals, have been reduced from $242.00 per day in 1938 to $79.20 in 1943. This is a result of continued and expected reduction in chemical dosages, which, as shown in Table 22, have been reduced from 3.17% of ferric chloride and 10.30% of lime in 1938 to 1.10% of ferric chloride and 3.12% of lime in 1943. This reduction of approximately $2.00 per dry ton for chemicals, when it is remembered that an average of about 35,000 dry tons are filtered annually, will result in a saving of considerable magnitude.

The statements of Messrs. Van Kleeck and Rudolfs concerning the practicability of using raw sludge as a fertilizer were particularly interesting to the writer in relation to present practice at the Minneapolis-St. Paul plant. Raw

sludge has certain objectionable characteristics including a possible health hazard. To obviate some of the difficulties presented in the use of raw sludge, this district has adopted the policy of allowing farmers to haul and use the filter cake only during the fall and early winter months from October to January, inclusive. No raw sludge is allowed to be hauled after February 1. As a matter of convenience the farmers apply the filter cake as it is hauled. The storage of the material, even for several days, creates odor problems and changes the characteristics of the sludge so that it is no longer workable. The use of fresh filter cake is not permitted on lawns and gardens within the city. Instead, the district stores filter cake for a period of two years, and this material (which is very workable and odor free) is made available for interurban use. Several years of experience have shown that at least two years of storage are necessary to place the material in a satisfactory condition for use. Shorter periods of storage result in a sticky foul-smelling mass.

This policy of the district, and the fact that, to the present time, the material is furnished without charge has resulted in a steady and rapid increase in the tonnage of material used as a fertilizer. The following are the quantities of filter cake used for such purposes:

Year	Tons
1939	21
1940	2,756.8
1941	3,678.3
1942	6,867.3
1943	11,580.3

Although use in 1943 represents only about 10% of the annual production of filter cake, there should be an even greater demand in future years.

In a very systematic and orderly manner, Mr. Mendelsohn gives an analysis of the operating experiences and improvements at the various plants. Such information permits some interesting comparisons.

The Symposium with its discussions has presented a rather complete picture of the field of sludge disposal. The tone of the discussions indicates that it has been thought provoking, and, if no other purpose was served, this alone would justify the presentation of the Symposium.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

UNUSUAL CUTOFF PROBLEMS—DAMS OF THE TENNESSEE VALLEY AUTHORITY

A SYMPOSIUM

Discussion

BY E. H. BURROUGHS

E. H. BURROUGHS,[12] M. AM. SOC. C. E.[12a]—The reader will be impressed by the bold scope and thorough execution of the deep cutoff construction on the Kentucky Dam, described in the interesting paper by Mr. Hays. The method used to provide an effective upstream water seal probably has salvaged an important dam site where the foundation materials would otherwise have precluded successful development of the relatively high-head structure to be supported.

The engineering and construction features of this structure should be emphasized, rather than the geology. A subterranean cutoff wall built by methods substantially the same as those used by the author will make valuable but defective reservoir sites suitable for development almost regardless of the geological conditions. Few changes are required to adapt this method to almost any type of difficult foundation, as practical experience has shown. The applicability of this method is not limited to cutoff walls built deeply into river beds where flowing ground water may be encountered, since the method is safe, rapid, and economical for almost any deep trench work, and the size of the working chamber can be reduced to an area in which the water inflow can be handled and sloughing controlled. The excavation principle is somewhat similar to the methods of middle-western contractors in the 1910's when they invaded the deep foundation field in New York, N. Y., with their so-called "Chicago method." With open caissons and heavy pumping equipment, plank-sheeted circular pits with steel-ring walers could be driven to great depths, for column footings, faster and cheaper in all soils than by the conventional methods of the pneumatic-caisson contractors.

NOTE.—This Symposium was published in November, 1943, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1944, by Berlen C. Moneymaker; April, 1944, by Portland P. Fox, and J. K. Black; and May, 1944, by L. C. Glenn.

[12] Pres., Ambursen Eng. Corp., and Vice-Pres., Ambursen Dam Co., Inc., New York, N. Y.

[12a] Received by the Secretary December 15, 1944.

As a last resort, where a water-logged material becomes too costly, slow, or dangerous to excavate, compressed air can be applied to the shaft until the difficult area has been traversed; but in no instance has it been needed.

In the case of the Hales Bar Dam, a cellular cutoff, built from the top downward, would have avoided heavy water losses and the eventual costly repair work described in Mr. Schmidt's paper. The writer can give similar instances, such as hydroelectric plants built under his direction on the porous limestone of the Shenandoah River in Virginia (one directly above the Luray Caverns) where solution channels swallowed enormous quantities of wasted grout.

It is difficult to conceive a cutoff wall or core-wall problem that will not be solved adequately by this type of construction. One of its greatest advantages, not mentioned by Mr. Hays, is that the men work without fear or restraint in the free air, as compared to conditions under the costly, laborious, and slow pneumatic-caisson method. Relatively little heavy equipment is needed. Safety for workmen against sudden inrushes of water or mud is assured by shafts of ample dimensions, where the undercut face of the excavation is only a few feet at most beyond shaft limits, as shown in Fig. 30. This strengthens the writer's view that the geological deficiencies of the foundation at Kentucky Dam are less important to the engineering profession than the designing and construction methods which repaired those deficiencies.

Once the initial drilling has determined the rock elevations at a dam site and has demonstrated suitability for the methods outlined, little is gained by further expenditure for an elaborate pattern of drilling, as any changes in the conditions are adequately met as the work progresses below ground.

Since 1928 the writer has applied the cellular cutoff wall design to four different projects, two actually having been built by that method. Each foundation material was entirely different from the others, and yet the advantages of the method applied with equal force to all four projects.

Designs were prepared under the writer's direction in 1930 for a high dam on the Kootenay River in British Columbia, Canada (later abandoned for financial reasons). The river bed was ledge rock but both sides consisted of rapidly rising, high banks of porous sand, gravel, and boulders, upon which large embankments were to be built as wings or abutments for the spillway. Core walls approximately 100 ft deep were required, but the cost of a sheet steel-pile core wall was prohibitive and driving it impossible. Comparable data for the cellular wedge type of core wall, built from the ground surface downward, showed a cost 46% of that of a concrete core wall of conventional type, constructed in a sheeted and shored open trench. This design reduced the top width of the excavation considerably and left adjoining material practically undisturbed. Any underground slumping of materials would be replaced by grouting as the core-wall construction progressed downward. Freezing was contemplated where necessary for controlling any water-bearing sand in the lower depths of the core wall, since the small confined area under each shaft was admirably suited to this method. Modern chemical soil consolidation would be equally helpful. The difficulty and danger of maintaining a deep sheeted and shored trench would have justified the method described even at an equal cost.

The writer believes the first example of the design and construction of a concrete cutoff wall built from the ground surface downward, making use of working shafts through the completed sections, was the Rodriguez Dam in Lower California, Mexico,[13] begun in 1928. This wall was constructed (without

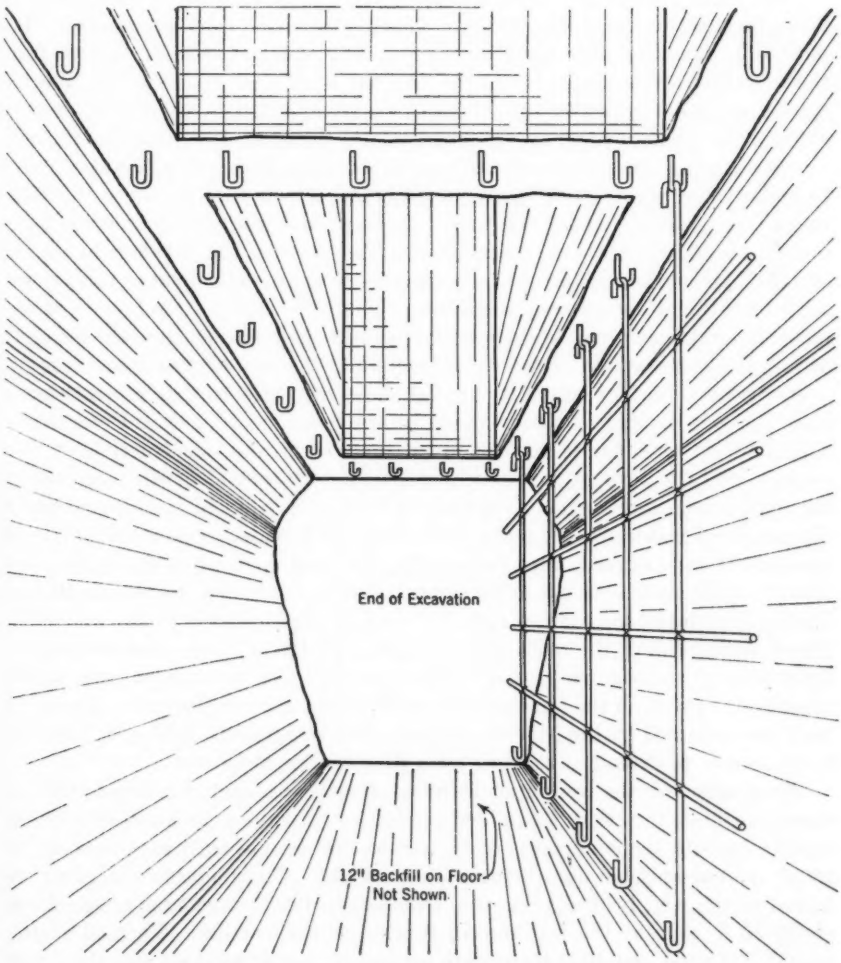

FIG. 30.—TYPICAL EXCAVATION FOR CELLULAR CUTOFF WALL BLOCK

reaching bedrock at the bottom) to a final depth of about 300 ft below the working level in the stream bed to provide the proper creep ratio for the head of 240 ft above river bed created by a full pond. The total distance or head from pond level to bottom of cutoff wall was 475 ft. This work was done at low cost by a small force without accident or serious injury to any workman,

[13] "Foundation Treatment at Rodriguez Dam," by Charles P. Williams, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 295.

against the full head of the Tijuana River. The structure was built through soft clay gouge, fugitive blocks of granite and ryolite, areas of crushed foundation rock, and lenses or continuous beds of sand of varying texture, including quicksand.

Mr. Hays is fortunate indeed to have had the opportunity of sketching the actual geology of the Kentucky Dam cutoff wall as the work progressed. His data afford an admirable check on the accuracy of the conclusions drawn from the wide network of borings. At the time the Rodriguez Dam site was selected for development in 1927, the Mexican Government was engaged in a very extensive irrigation program elsewhere in the republic and little money was available for a boring program. However, even had this government been able to make an advance subsurface survey as extensive as that made for the Kentucky Dam cutoff (which the writer understands cost about $500,000), the problem would not have been simplified. The site was the only one available, and the Mexican Government was committed to the development of this project for international and economic reasons. The writer cannot see how even the most elaborate pattern of borings would have affected, measurably, the ultimate design and construction of the cutoff wall and other foundation structures.

Only nine borings and fifty-two shallow test pits (which represented a cost of about $7,500) had been made by the National Irrigation Commission when the writer's company took over the design and construction of this project. Of the borings, five indicated satisfactory rock at depths not exceeding 50 ft, but no cores could be obtained from four, two of which indicated badly disintegrated materials. A main fault about 30 ft wide and a secondary fault about 10 ft wide existed, filled principally with clay gouge, but also with many pockets or layers of water-bearing sand. The area between the two faults was filled with crushed foundation rocks of varying sizes, bedded in clay and sand. The hanging wall of the main fault had been shattered for a considerable distance beyond the edge of the fault, necessitating special treatment, and similar conditions existed on the opposite side of the filled-in river channel. The main fault was assumed by the Mexican Government's consulting geologist, Frederic L. Ransome, to extend downward for a thousand or more feet.

Excavation for the Rodriguez Dam cutoff wall was begun in an open trench, curved in plan to secure the added strength of a subterranean arch without interfering with the connection at the top to the straight upstream deck of the 240-ft Ambursen type dam being built above it. Considerable sloughing occurred in the wet, broken formation resulting in the change to a wedge shape about 25 ft wide at the top and 15 ft wide at the bottom. Vertical shafts, mostly 5 ft wide and 10 ft long, were formed in this excavation and the entire trench plug was then concreted. The wedge shape was expected to aid in supporting the block against settlement, which would halt itself by consolidating the material on each side, but no measurable settlement ever occurred.

When the shaft excavation was well under way, construction began on the main dam itself, so that work on the dam was not delayed pending completion of the cutoff.

The vertical cutoff arch was projected into the fractured hanging wall by a system of grouting from the surface and from adits (as shown in Fig. 31), after

other adits, forming a continuous vertical wall, had been cut into the hanging wall from the adjoining shaft to the distance considered economical by that method, as compared to the cost by drilling and grouting. Section C–C of

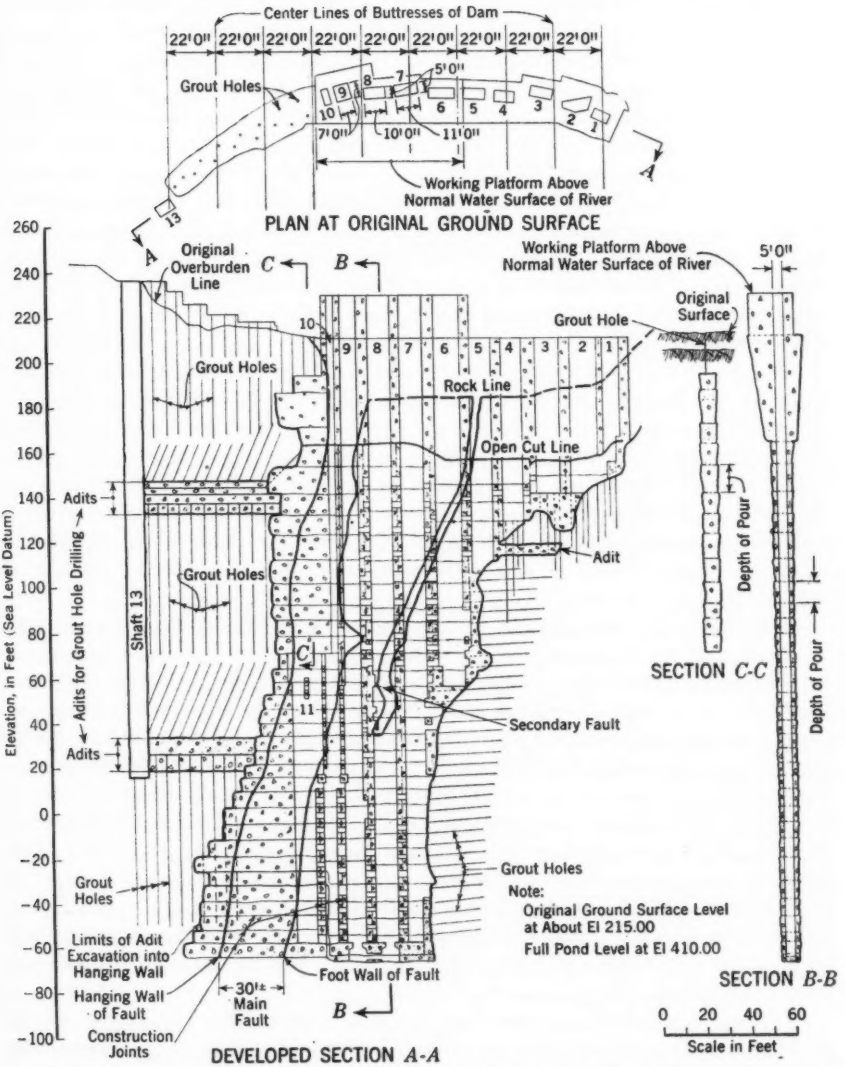

FIG. 31.—RODRIGUEZ DAM: PLAN, AXIAL SECTIONS, AND VERTICAL SECTIONS, SHOWING PROCEDURE

Fig. 31 shows the novel method used for constructing these adits. The wedge-shaped adit at the highest elevation was first excavated and concreted. Then the adit next below was similarly excavated, using the bottom of the concrete next above as a roof, which made for very rapid and safe excavation. Much

grout was taken by the drill holes in the hanging wall, which supports the right wing abutment of the main dam.

The writer doubts whether, on a project where the utmost economy might have to prevail, the cost of the author's method of redrilling 36-in. and 48-in. holes through the finished concrete blocks—to enable the cutoff wall to be deepened progressively below and concreted by the use of these very restricted openings as working shafts—would be justified. It has been the writer's experience that, by forming large rectangular shafts, plenty of room was obtained for the rapid evacuation of excavated material and for pipes, conduits, telephone cables, etc. More important still, where water-bearing sand and porous crushed rock under great pressure invited slumps and blowouts, the generous cross-sectional area of the working shaft (about 5 ft by 10 ft) enabled the men to work with greater confidence, and hence more effectively, than if the exit from the working chamber had been a circular shaft of small diameter. Wide steel rungs were set as a continuous ladder in every shaft from the top to the bottom —an emergency exit for the workmen.

The lengths of 6 ft along the axis, used for the Kentucky Dam cutoff wall blocks where the drill hole was 36 in., and 7 ft where it was 48 in. in diameter, called for mining out about 18 in. under the sides of each block—practically the average undercut for the Rodriguez Dam. However, the ultimate cost of the job is controlled more by the actual number of shafts and blocks than by their dimensions; hence, the writer believes that a given length of cutoff wall using long blocks with big shafts, similar to the Rodriguez cutoff, would cost much less than a length of wall with double the number of blocks half as long with their smaller shafts.

Attempts were always made to impart a wedge shape to the excavation for each block before concreting. The writer's preference for this procedure comes from the conviction that wedge-shaped pours are subject to less danger of settlement and that they form a tighter and stronger structure. They also create a longer and rougher path for percolation; but this was not taken into consideration in computing the creep ratio. These individual blocks are actually given support in four ways: (*a*) From a vertical reinforcing system; (*b*) by friction on the sides of the excavation; (*c*) by the horizontal steel reinforcing bonds, and (*d*) by bridging or beam action from the contacts with the adjoining blocks at the same level. The wedge-shaped configuration gives a fifth support. Faster progress was made in excavation by leaving a layer of 10 in. or 12 in. of loose material on the floor of the excavation and by pouring the concrete block on it, since excavation for the next lower block was made easier by starting from this soft loose layer.

The writer's experience was identical with that of Mr. Hays—by driving alternate shafts or blocks first and maintaining these ahead of the intermediate blocks, the width (thickness) of the latter could be considerably reduced and lifts could be extended deeper than for the adjoining blocks and with less pumping. Only two faces had to be maintained, as against four in the shafts which were pushed ahead first.

Straight bars were first used as the vertical reinforcing, and, as at the Kentucky Dam, were driven or drilled into the bottom of the excavation before

the concrete was poured. Later, hooked bars were found more convenient. As shown in Fig. 30, the hooks made for better working conditions than a series of long bars projecting down into the area being excavated. The horizontal bars were driven from the open excavation laterally into the intermediate unexcavated area, as was the case at the Kentucky Dam.

At the beginning there was no certainty that this novel cutoff wall method could be followed to the bottom of all shafts, but, as ample compressor capacity was available, the job was always in readiness to switch to the pneumatic process, which never had to be adopted. Even areas of semifluid quicksand were usually controlled without undue delay by pushing adjoining shafts down past them. Drainage through the walls of these intermediate blocks soon eliminated the difficulties in the bad pockets.

It is not clear to the writer whether the 36-in. and 48-in. drill holes at the Kentucky Dam through which the excavation and concreting were conducted were eventually filled with concrete or spoil, or whether they were left open and merely capped at the top. Likewise, the details of the connection of the steel sheet-pile core wall of the dam to the cellular concrete core wall would be of considerable interest. Upon completion of the Rodriguez cutoff wall, all but two of the shafts were backfilled with concrete. However, it was considered that it might be desirable at some future time to extend the cutoff wall deeper, should seepage beneath the cutoff occur in dangerous quantities. Therefore, in shafts 8 and 9 a cylindrical form 48 in. in diameter was installed, the remainder of the shaft being filled with concrete. These two circular shafts were then plugged with concrete at the bottom, backfilled with sand and gravel, and topped with a specially designed watertight cover. In the future if it is considered advisable to deepen this cutoff, steel cylinders will be attached to the two circular shafts and excavation and concreting operations will be conducted through them from a point above the lake level.

It would be interesting to know how much grout the vertical joints between the Kentucky Dam cutoff blocks took and whether any attempt was made to grout the horizontal joints between the blocks. At Rodriguez Dam grout pipes were built into all contacts between concrete blocks and into the outside foundation material adjoining the cutoff wall. Practically no grout could be forced into any of the joints, indicating no subsidence of the blocks, probably due to their wedge-shaped formation and reinforcement. The pipes to the exterior of the blocks were capped on the inside when installed but were left open to the outside ground. When excavation began below, the caps were replaced by canvas hose, which led the outside drainage freely into the sump. This drained off much outside water that otherwise would have caused difficulties. To avoid building up pressure, the pipes were left open until several sections of the concrete had been placed below them, when grout was forced out into the foundation material on both sides of the wall, usually under a pressure of 100 lb per sq in. Every precaution was taken to reduce the head that could build up against any part of the excavation. Many slumps inevitably occurred behind the blocks as excavation progressed, but always were stopped by quick plank shoring. For many blocks, some sheeting and bracing were also necessary to restrict the amount of excavation. The sheeting and bracing were of

the most rudimentary nature possible, usually merely a few short planks braced as might be most convenient. Most sheeting was left in place and, it is assumed, became subsequently embedded in the grout.

Contrary to Mr. Hays' experience on Kentucky Dam, where the water leakage increased as the work deepened, the Rodriguez Dam inflow at 200 ft to 300 ft below river level was no greater than at 50 ft, and as far as could be determined had no relation to the depth of the shaft. Rather, it depended on the texture of the material through which the wall was passing. The drainage program did much to ease the water control, and, by keeping as many drains open as possible above the working chamber, little or no trouble was experienced in maintaining satisfactory pumping from the sumps. Relay pumping stations were installed whenever needed, the water usually being pumped into a small tank with a high-head electric pump over it. The tank and pump were both placed in one side of the shaft where they would not disturb the hoisting of muck. Pipe lines, 4 in. in diameter, were originally installed in each shaft through which the water was pumped to the surface, but, as confidence was gained in the method being pursued, adjoining shafts were interconnected, which expedited the work materially and considerably reduced the pumping installations.

The over-all cost per cubic yard of work in the shafts was less than that for the adits cut into the hanging wall of the fault, and, as the hanging wall was departing from shaft 10 when the cutoff wall was about halfway down, shaft 11 was opened under one of the adits, which shortened all the remaining adits. Shaft 11 was interconnected with shaft 10 and was worked entirely from the latter shaft.

To maintain a continuous record under full reservoir conditions of the pressures at the bottom of the cutoff wall and under the dam itself, instruments including pressure gages and pipes for soundings were installed in the dam foundations and in the sides and bottom of the shafts before concreting. Frequent observations have indicated that such leakage as can be detected in the river bed below the dam does not come from beneath the cutoff wall, but through seams in the earthquake-shattered banks of the river which carry the side wings of the dam, below the limits of the grouting done in the bottom of the conventional relatively shallow cutoff wall where the dam rests on rock. No lateral or longitudinal flow can be detected along the base of the dam, and there is no evidence of any water passing under the cutoff wall at its bottom. At one point along the bottom of the cutoff wall the hydrostatic pressure is greater immediately downstream from the wall than immediately upstream.

The National Irrigation Commission felt that, by applying one of the conventional creep-ratio theories to the material in the fault, it would be safe to stop the cutoff wall at some depth between 200 ft and 250 ft. The decision to extend it to 300 ft was influenced by the low cost and relative ease of the cellular cutoff wall method.

The cellular cutoff wall method (including the features of building the cutoff wall from the top surface downward through shafts instead of from the bottom up) and also the adit construction method used in the firmer rock areas (of building a top adit first in wedge shape to form a safe and convenient working

chamber below it for each succeeding wedge-shaped adit, thus constructing downward a continuous concrete curtain or cutoff) were developed by Spencer W. Stewart, M. Am. Soc. C. E. To the best of the writer's knowledge the Rodriguez Dam was the first project on which these principles were ever used.

A third project using designs and construction methods similar to those described in Mr. Hays' paper (except as to the drilled shafts) was the Bristol Dam on the Pemigewasset River in New Hampshire. This work was incidental to adding 30 ft to the height of the dam in 1931, to increase the head on the power plant from 50 ft to 80 ft—for which program the Ambursen type dam had been adopted ten years earlier. Deep core walls were constructed adjacent to the abutments on each side of the river, the maximum depth being 113 ft, without lowering the pond level or shutting down the power plant. By the method adopted the core walls were constructed with no difficulties, although they were extended 77 ft below the pond level.

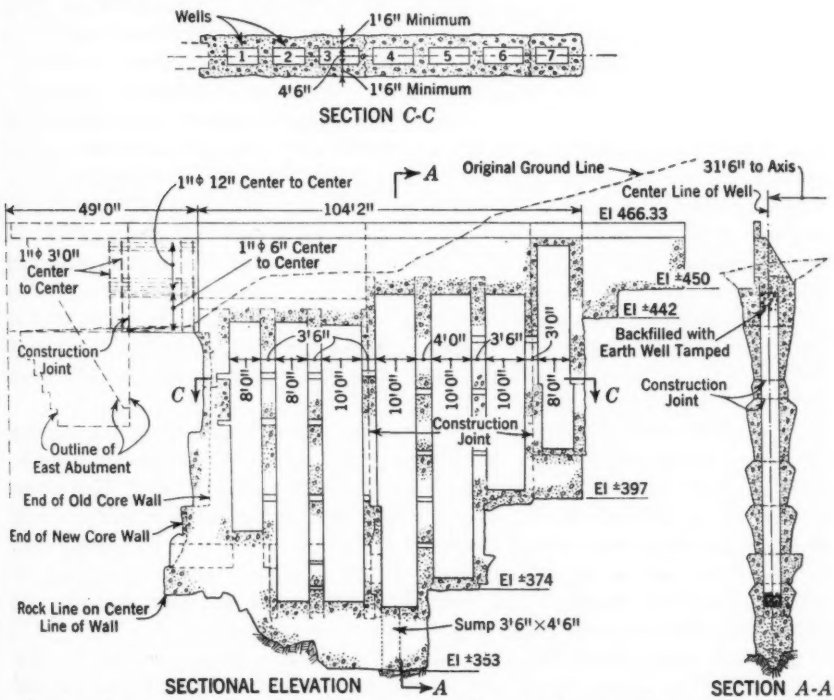

FIG. 32.—BRISTOL DAM: PLAN, AXIAL SECTIONS, AND VERTICAL SECTIONS, SHOWING PROCEDURE

In constructing the Bristol core wall, an open trench 14 ft wide and 16 ft deep was first excavated (see Fig. 32). The sides were roughly sloped, so that the cellular masonry section would be partly supported by wedge action when material was excavated below it.

Because of experience gained on the Rodriguez Dam and in view of more favorable materials at the Bristol site, the shorter core wall in the right bank

was built as a single block for its entire length, but contained three vertical shafts. The longer core wall behind the left abutment was built in three sections, two with three working shafts apiece and the other containing one. When the cellular concrete core wall was completed, a concrete bottom slab was poured in all shafts, and all were backfilled with spoil from the excavation.

Practically no grout was taken between the vertical and horizontal concrete surfaces of the blocks and very little was taken in the areas outside the walls.

Although no attempt was made to speed up the construction or to set a record of any kind, both core walls were built in 95 days, a demonstration of the advantages of this construction method.

Steel sheet piling could not have been driven to the necessary depth in the Bristol material. Estimates for conventional open-trench methods, including sheeting and pumping, indicated a total over-all cost for the excavation of $10 per cu yd, whereas the final cost on a comparable basis for the method followed was $4.15 per cu yd. One reason for the low cost was the freedom with which the men worked as compared with the caution necessary when working under adverse conditions in a deep heavily braced and shored trench.

The core wall for the Quabbin Dike Dam in Massachusetts, for which twenty-five bids were received based on the pneumatic-caisson method in 1933, affords persuasive comparative data as between the cellular core-wall free-air method (built from the ground surface downward) and the pneumatic-caisson method. Earth excavation through the caissons was estimated at 33,000 cu yd, and the depth averaged about 80 ft. The three lowest bids averaged $7.50 per cu yd; and all twenty-five bids, $12.67. Moreover, the heavy pumping and grouting costs were paid for under other items, and vastly more grouting was involved than with the cellular type of core wall. Rock excavation cost must always be higher because of the necessary use of expensive sand hogs instead of common labor, and concrete costs would also be somewhat higher under the pneumatic method.

The useful field for the method of cellular construction from the surface downward as described in Mr. Hays' paper is not limited to foundation materials of a soft or porous nature such as those at the Kentucky, Kootenay, Rodriguez, or Bristol dams. This method is likewise entirely suitable and surprisingly economical where shattered and seamy rock is to be penetrated, such as is the case at many California and other western sites. Naturally, it cannot be applied where the rock is well bedded or where conventional grouting will tighten the fractured material and restore sound bedding.

In 1930 when construction of the gravity section masonry dam for San Gabriel Site No. 3 was abandoned by the Los Angeles County Flood Control District because excavation showed the rock to be incapable of carrying a dam of that type safely, preliminary designs and estimates were prepared by the writer for an Ambursen type spillway dam, which, because of its relatively low base pressures, was entirely suited to the foundations. However, the problem of supporting and tightening the high rock banks on each side of the river was difficult to solve. The high rock banks were bad in their natural condition, and heavy blasting had further impaired their stability. An abutment was designed for each side of the spillway and three parallel cellular core walls were

laid out for each, extending at right angles to the abutment. The upstream and downstream walls were to be sunk only enough in depth and in length to confine and support the shattered and seamy rock as found necessary, and the center core wall on each bank was to be constructed 200 ft down to cut off percolation through the banks. The cost by the method under discussion, starting at the top and working downward to the desired elevations, was found to be very much less than for core walls built in an open trench, which method probably could not have been used at all. As the subsequent choice of the district was for a different site, the cellular design was not projected to the construction stage, but its speed and low cost in poor rock formation were thoroughly established.

Mr. Hays' paper is a very instructive recital of the methods followed in solving a difficult engineering problem. It serves to emphasize that the cellular type of construction for cutoff walls and core walls has proved a practicable and efficient method of solving the problems incidental to providing an adequate water seal for poor foundations. The writer's experience with the aforementioned four projects shows also that this method can be used at low cost. Not only because of this economy, but because of the safety of the method itself, the cellular cutoff wall type of construction, from the ground surface downward, makes available for rapid and economical development many dam sites previously rejected as unsafe or uneconomical because of foundation difficulties. Mr. Hays has also added much knowledge to the scant information available to the engineering profession on the subject of compacting yielding material by grouting. The paper is a fine contribution to a relatively new and little understood field.

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DISCUSSIONS

CHARACTERISTIC REDUNDANTS USED FOR ANALYZING STATICALLY INDETERMINATE STRUCTURES

Discussion

BY FREDERICK S. MERRITT

FREDERICK S. MERRITT,[26] JUN. AM. SOC. C. E.[26a]—Although the purpose of this paper was to present some of the principles involved in utilizing orthogonal forces, the value of the paper as a contribution to structural theory suffers because:

(1) The terminology involved is awkward;
(2) The visualization of the physical processes signified by each step in the calculations is difficult, if not impossible;
(3) In addition to requiring the computation of constants which are used in other well-known methods, this theory requires the solution of simultaneous equations plus the determination of the roots of an *n*th-degree algebraic equation; and
(4) The one advantage claimed (after the characteristic forces have been found, redundant stresses may be evaluated without the further solution of simultaneous equations) is not exclusive.

The idea underlying advantage (4), however, is of extreme importance to structural theory and deserves further elaboration.

Whether this method of analyzing statically indeterminate structures, using orthogonal forces, can be improved so that it can stand on equal terms with procedures such as the "Dummy Unit Load Method" is questionable. On the other hand, the principles outlined in this paper may be a start in the right direction, especially if the subject of characteristic redundants is approached from another angle than that of orthogonal forces.

(1) A change in viewpoint may improve the terminology. The use of such words as "characteristic forces," "normalizing," and "orthogonality," although

NOTE.—This paper by John B. Wilbur, Assoc. M. Am. Soc. C. E., was published in June, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1944, by Charles A. Ellis; and December, 1944, by Ralph W. Stewart, Myle J. Holley, Jr., M. W. Huggins, and Leon Beskin.

[26] With *Engineering News-Record*, New York, N. Y.

[26a] Received by the Secretary December 18, 1944.

mathematically expressive, is discouraging from a structural viewpoint, because certain physical properties are implied which, unfortunately, have no connection with the structure being analyzed. Actually, an abstract mathematical operation is involved. If these terms were more closely related to structural action, their use would not be so awkward.

(2) A different approach may correlate, to a greater extent, the conception of characteristic redundants with the structural action involved. Such correlation is a desirable goal in all structural problems since it provides a continuous check on computations; but it is a goal that cannot always be achieved. As the method is presented, a mass of computation is required to determine A_{mn}, a quantity which, although it is essential to this method, has no immediately perceivable relation to any physical action in the structure. If a relation can be established, the principle involved in using orthogonal forces would be simplified considerably. After a study of the formulas for normalizing and orthogonalizing, the question naturally arises as to whether the concept of vectors and tensors could establish such a desirable relationship. For example, Eq. 10 indicates that Q_{nm} could be the component of a unit vector $\mathbf{Q_m}$, and, thus, Eq. 14 is the vector equation, $\mathbf{Q_m} \cdot \mathbf{Q_n} = 0$, $n \neq m$. If $\mathbf{Q_m}$ could be shown to have some meaning structurally and if the participation factors are related vectorially to $\mathbf{Q_m}$, a clearer picture of the principle involved in this method would result.

(3) By approaching the problem in a different manner, the solution of simultaneous equations may possibly be avoided or the number of equations may be considerably decreased, much as moment distribution methods have succeeded in doing for continuous, statically indeterminate beams and frames. A vectorial approach, involving the selection of suitable axes to simplify the resolution into components of a vector such as $\mathbf{Q_m}$, may be indicated. However, there are other possibilities in methods not involving characteristic forces.

(4) A change in the method of analysis will not spoil the advantage that, once characteristic forces have been found, redundant stresses may be evaluated without the further solution of simultaneous equations. Almost all the well-known methods possess this advantage because the equations may generally be written so that the coefficients of the unknowns depend solely on the structural framework and are independent of loads or distortions.

However, this advantage is not to be belittled. For example, the popular method of fixed-end moment distribution (which, in general, does not require the solution of simultaneous equations) requires that whenever the load is changed either influence computations must be made in the first place or the structure must be analyzed again. In schoolroom problems, this may not be much of a handicap, but in practical design, where the stresses resulting from dead load, live load, wind forces, longitudinal and lateral forces, earth pressure, earthquake, temperature, shrinkage, and settlement must be determined separately, reanalysis is a big handicap and characteristic redundants are definitely advantageous.

The characteristic redundants need not be orthogonal forces, or even forces at all. This is suggested in the paper by the alternate basis for orthogo-

nality, in which forces and deflections are involved. Deflections alone could be used. Whether or not in the general case the use of deflections would simplify the problem will have to be determined. Such a possibility is indicated by the use of end rotations in the solution of problems involving continuous, statically indeterminate beams and frames.

For example, in the method of moment distribution, by means of moment ratios, advantage is taken of the fact that the ratio of the moment induced at one end of an unloaded span of a continuous structure to the moment "carried over" to the other end is a constant independent of the magnitude of the moments. This ratio is a characteristic redundant in the same sense as the characteristic forces described by Professor Wilbur. By using it, the solution of simultaneous equations is reduced to a minimum or entirely eliminated. Offhand, the moment ratio in a given span might be considered a sort of characteristic force, but, actually, it is a function of the end rotations of the unloaded span considered as simply supported but subjected at either end to a unit torque.

The distribution of an unbalanced moment at a joint to the several connecting members is a function of the moment ratios. Also, in a loaded span, the end moments produced by the loads are a function of the moment ratios and of the end rotations due to the loads when the span is considered as simply supported. Thus, all the elements necessary for the solution of a continuous beam or frame problem are present, when the moment ratios are known.

Are there characteristic redundants in the form of deflections that can be used, in a similar manner, to determine the distribution of a force to each of several structural members which support it at a joint? If so, the problem of analyzing statically indeterminate structures should be approached from a viewpoint using these characteristic redundants.

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DISCUSSIONS

CONSTRUCTION OF THE FLOW NET FOR HYDRAULIC DESIGN

Discussion

BY J. W. HOWE

J. W. HOWE,[4] M. AM. SOC. C. E.[4a]—The amplification of a graphical method for checking the location of streamlines in a flow net, as presented by the author, offers detailed working directions which should prove useful to any one desiring to make such a check. Experience, patience, and no little skill are essential in drawing a correct network, and even with these the labor may be considerable. The graphical checking of the tentative location of streamlines is also time consuming and probably offers little advantage to an experienced hand. To the inexperienced, however, such methods, no doubt, will provide a somewhat more rapid approach to a solution, since an essentially correct location at one point in the network is of material aid in shaping the pattern for a considerable distance in either direction.

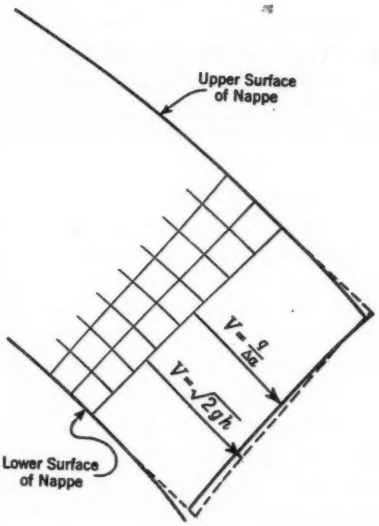

FIG. 6

The flow net has proved useful in many situations involving two-dimensional flow which is essentially irrotational. Its application to three-dimensional flow is subject to question, however. The writer is of the opinion that the accuracy of such an application would vary directly with the departure from two-dimensional conditions.

NOTE.—This paper by H. Alden Foster, M. Am. Soc. C. E., was published in May, 1944, *Proceedings*.

[4] Prof. and Head, Dept. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

[4a] Received by the Secretary September 29, 1944

In the experimental work done by Professor Camp (8),[4b] only the discharge and the position of the nappe surfaces were determined; pressure measurements in the nappe were not made. Because of the radial convergence of stream filaments in passing over the circular weir there must be pressure in the nappe even in regions where the streamlines of the two-dimensional flow net are parallel.

The writer applied the graphical method to a section downstream from a circular weir (head-diameter ratio, 0.14) at a point where the streamlines were practically parallel (such as point E, Fig. 4). The boundaries of the nappe were taken from Professor Camp's experimental data, and the location of streamlines was verified by the author's graphical method. The cross-sectional areas of the stream filaments were determined, and from these and the discharge, the mean velocities were computed. If the flow were two-dimensional, no pressure could exist in the nappe at this section and the velocity at any point would equal $\sqrt{2gh}$. A comparison of the latter quantity with the velocity obtained from the flow net is shown in Fig. 6. There is an indication of some pressure in the nappe although it does not reduce to zero at the underside as it should. Since the degree of convergence was quite small in this example, the indicated pressure may be of about the right magnitude. However, experimental substantiation of this possibility would be required to establish the validity of the flow net in such a case.

Bibliography.—

(8) **"Tests of Circular Weirs,"** by Cecil S. Camp and J. W. Howe, *Civil Engineering*, April, 1939, p. 247.

[4b] Numerals in parentheses, thus: (8), refer to corresponding items in the Bibliography (see Appendix I) at the end of the paper, and at the end of discussion in this issue.

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DISCUSSIONS

STRESSES IN THE LININGS OF SHIELD-DRIVEN TUNNELS

Discussion

BY J. A. VAN DEN BROEK

J. A. VAN DEN BROEK,[13] M. AM. SOC. C. E.[13a]—Mr. Bull's impressive paper deals with different subjects, three, to be exact—soil mechanics, elasticity, and theory of strength—all in relation to his main topic, that of lining stresses in shield-driven tunnels. The writer wishes to confine his comments to only one of these subjects, that of theory of strength, in Sections 4 to 12, inclusive.

For some fifty years or more designers have been familiar with the fact that an elastic curve, suitably interpreted, is an influence line. Much depends, however, on the "suitable interpretation." The use of elastic curves as influence lines for closed rings is new, at least to the writer, and in itself constitutes a worthy contribution to the theory of strength. In addition to presenting a difficult subject simply, lucidly, and in very readable language, Mr. Bull has done a vast amount of work, very little of which appears on the surface. All his computations, which must be voluminous, are on file in the Engineering Societies Library.[4a] This is eminently proper. Had they been published few would have checked them, and even then few would have been absolutely sure of the results. Mr. Bull offers tables which are of the greatest value. They are classic in the sense that they seem to provide the last word in regard to the analyses of circular rings of uniform cross section.

Mr. Bull has apparently neglected one thing, inclusion of which some of his readers would have appreciated. He has failed to call attention to some obvious partial checks of his tables which appear readily at hand. This may be understandable. Apparently Mr. Bull set himself the task of condensing enough material for a book within the space of only thirty pages. No doubt he applied numerous checks as he progressed. Possibly he regards some of them as self-evident.

NOTE.—This paper by Anders Bull, M. Am. Soc. C. E., was published in November, 1944, *Proceedings*.

[13] Prof., Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.

[13a] Received by the Secretary December 1, 1944.

[4a] 29 West 39th St., New York, N. Y.

The writer has applied Mr. Bull's tables to the solution of certain problems, the answers for which were known in advance. This, to him, serves to inspire confidence both in Mr. Bull's logic and in the correctness of the values given in the tables. Not that Mr. Bull's logic needs confirmation. On the contrary, it is as clear as a crystal and rings true. Especially, Section 10 seems a master stroke. If Mr. Bull will realize that the writer has full faith in his own logic, yet questions all his own computations, he will pardon him, he trusts, when he refuses to accept any tabular data on faith. Typographical errors sometimes occur, and, if tables are to stand the test of time, a great deal of independent checking appears justified.

Below are offered a few such simple checks:

1. The formula for the bending moment in a ring subject to a number of equal and evenly spaced radial forces P is

$$M_A = \frac{P\,r}{2}\left(\frac{1}{\sin\phi} - \frac{1}{\phi}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

in which ϕ is one half the angle between the forces. If this formula is applied to Fig. 4, with $\phi = 11° 15'$, $M_A = 0.0164\,P\,r$. Adding Col. A_{Mr}, Table 1 (without including the two terms -0.00822—apparently Mr. Bull's solution for the same problem), the result is also $M = 0.0164\,P\,r$. To the writer, this constitutes one partial check. Mr. Bull's correction factor of 0.00822 seems sound and is very ingenious indeed.

2. The addition of all the entries in Col. A_{Mr}, Table 1, including the two correction factors -0.00822, gives zero for an answer. This was explained in the text and is somewhat of a check. When it is considered, however, that Table 5 was obtained from Table 1, that it involves a regrouping of the constants in Table 1 nine separate times, and that each time the columns for moment reactions caused by radial loading add up to zero, then it is hard to conceive how any typographical or other error may be hidden without being detected.

3. As a third independent check that the values in the tables are numerically correct, apply them to the evaluation of the moment at the bottom of a circular pipe of constants E and I, flowing full with water and supported radially by two reactions, each 33° 45′ removed from the vertical diameter and therefore corresponding to P_2 and P_{15} in Fig. 4. The so-called exact solution may be obtained by superimposing Case V and Case VIII[14] in the writer's book, "Elastic Energy Theory." This solution involves the assumptions of elasticity, constants E and I, and constant circular shape which are the same assumptions as those involved in Mr. Bull's theory.

Substituting $\phi = \pi$ in Case V, $M_A = +0.23873\,W\,r$, in which W is the weight of the water.

Substituting $\phi = \pi$ and $\phi_1 = \frac{3}{16}\pi = 0.58905$ radians in Case VIII,

$M = -0.23919\,W\,r$. Superposing these two values, $M_A = -0.00046\,W\,r$.

[14] "Elastic Energy Theory," by J. A. Van den Broek, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1942, pp. 128–129.

This value checks the value shown[15] in the right half of Fig. 14, in which M_A is shown as substantially zero for the case of $\phi_1 = 33° 35'$.

In evaluating, by means of the tables, the moment at the bottom point of a ring (point A, Fig. 4) loaded with hydrostatic pressure and concentrated radial reactions R applied in the middle of Sections 2 and 15, Fig. 4, the value of the P-forces was computed as follows: The hydrostatic pressure, say, at the midpoint of segment 6, is $w\,r\,(1 + \cos \phi_6) = w\,r\,(1 + \cos 123° 45') = 0.44443\,w\,r$

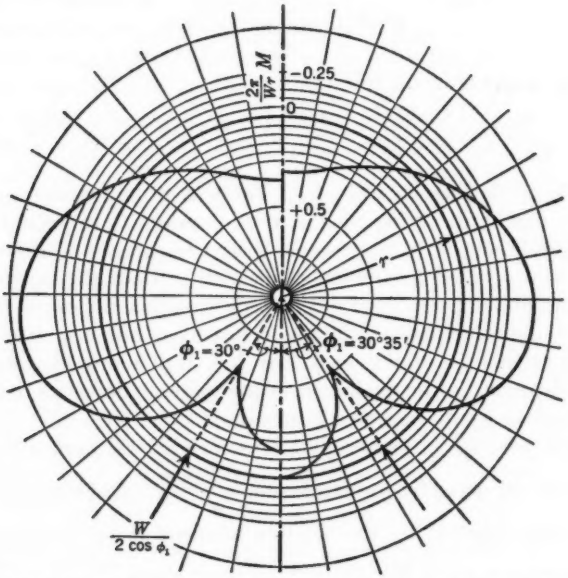

FIG. 14.—MOMENT DIAGRAMS, SYMMETRICAL ABOUT CENTER LINES, FOR CYLINDRICAL SHELLS FLOWING FULL WITH RADIAL SUPPORTS

(the relation between the unit weight of water, w, and the total weight of water, W, is $W = \pi\,r^2\,w$). Considering this pressure to act uniformly over segment 6, $P = 0.44443\,w\,r \times \frac{\pi\,r}{8} = \frac{0.44443\,W}{8}$. At segment 2 the reaction R is of opposite sense to that of the P-forces and equals $\frac{W}{2\cos 33° 45'} = \frac{W}{1.66294}$. The hydrostatic force P_2 acting on segment 2 would be $w\,r\,(1 + \cos 33° 45') \times \frac{\pi\,r}{8} = \frac{1.83147\,W}{8}$. The resultant force acting on segment 2 would be $R - P_2 = 0.372410\,W$. The other P-forces are computed similarly. Multiplying these values for P-forces, thus obtained, by their corresponding values listed in Col. A, Table 5, for moment reactions caused by radial components, and adding this entire column, with due regard for signs, $M_A = 0.000446\,W\,r$. This is in substantial agreement with the value of $0.00046\,W\,r$ obtained by the so-called exact analysis.

[15] "Elastic Energy Theory," by J. A. Van den Broek, 2d Ed., John Wiley & Sons. Inc., New York, N. Y., 1942, p. 136, Fig. 57.

4. If a circular pipe full of water, supported at its bottom, point A, is considered, force P_2 is included in the tabulation and the reaction R is disregarded. Table 5 then yields $M_A = 0.23877\ W\,r$, which compares with the value of $M_A = 0.23870\ W\,r$ obtained by the so-called exact analysis developed previously.

5. Fig. 15 represents a ring subject to sixteen uniformly distributed tangential forces P. The equation for the moment in this ring, at any point, is

$$M = \frac{T}{2\,\pi}\,(\phi - 2\sin\phi) \qquad (25)$$

Thus, at point A the moment is

$$M_A = \frac{16\,P\,r}{2} = 8\,P\,r \qquad (26)$$

Adding all the coefficients in Col. A_{Mt}, Table 1, provided they all are given the same sign, and multiplying by $P\,r$, the moment M_A is 7.99999 $P\,r$, which again is a beautiful check. As to signs, the writer knows no rule in regard to elastic curves interpreted as influence lines other than the one which states that, when force and deflection are of the same direction and sense, their product is positive. When they are of the same direction but opposite sense, their product is negative. It would seem that Mr. Bull followed some other rule which does not appear to be explained in the paper. Some more light on this very crucial matter would be much appreciated.

6. The shear stresses in an aeroplane fuselage act tangential to the surface (see Fig. 16) and are given by the expression $\frac{W\sin\phi}{\pi\,r\,t}$, in which t is the thickness of the ring. The equation for the bending moment at any point [16] is

$$M = \frac{W\,r}{2\,\pi}\left(1 - \phi\sin\phi - \frac{\cos\phi}{2}\right) \qquad (27)$$

Therefore,

$$M_A = \frac{3\,W\,r}{4\,\pi} = 0.2387\ W\,r \qquad (28)$$

The shear stress at the midpoint of segment 6, for example, is $\frac{W\sin}{\pi\,r\,t}$ $(5\frac{6}{7}° 15')$ $= \frac{0.82165\ W}{\pi\,r\,t}$. Assuming this stress to act uniformly over the entire segment,

$$P_{6t} = \frac{0.82165\ W}{\pi\,r\,t} \times \frac{\pi\,r\,t}{8} = \frac{0.82165\ W}{8} \qquad (29)$$

Computing the other P_t forces in similar manner, multiplying them by the coefficients for A in Table 5(*b*), and adding the resulting numbers, $M_A = 0.2387\ W\,r$.

These few checks have greatly increased the writer's confidence in the accuracy of Mr. Bull's tables. Possibly they may serve the same purposes for others.

[16] "Elastic Energy Theory," by J. A. Van den Broek, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1942, Appendix II, p. 271, and Example 31, p. 123.

Mr. Bull nominally crowds enough material for a book into a limited number of pages, dealing strictly with but one issue—tunnel stresses. To the writer he has done a great deal more than that. He has offered a sound bit of engineering philosophy. Section 8 is clever, and Section 10 is a stroke of genius. He has offered a very simple method which has applications far beyond tunnel analyses. Aeroplane fuselages, dirigible airships, submarines, large pipes, and various kinds of tanks are all closed rings. Last but not least, there is the large size sewer. The analysis of a sewer on the assumption that it may be regarded either as a fixed arch or as a hinged arch is so palpably beside the truth as to be worthless. Mr. Bull, it seems to the writer, offers a method which lends itself to any degree of precision. Mr. Bull has chosen to compute the mathematical values found in his tables to an accuracy of five significant places. All this was natural and necessary. The accuracy of five

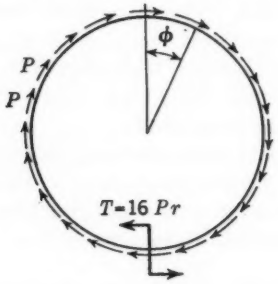

FIG. 15

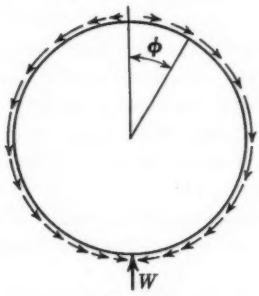

FIG. 16

significant places is fictitious, of course. It was necessary, however, if Mr. Bull wanted to apply checks such as the foregoing and, furthermore, if he wanted some of his columns to add up exactly to zero. The procedure is also well justified in view of the fact that Mr. Bull's tables may be used several generations hence in their present form. However, to apply Mr. Bull's method of analysis to a sewer section with a variable moment of inertia, and other than perfect circular outline, would require about double the amount of work represented by Mr. Bull's complete paper in the Engineering Societies Library. Thus, although possible and accurate, it may not be feasible. Furthermore, such work could probably not be generalized as Mr. Bull has done for the circular ring of constants E and I and would have to be repeated for each new section considered in any one design problem. One way out of this dilemma, it seems to the writer, would be to obtain the elastic curve experimentally instead of mathematically. This would not result in an accuracy such as is implied in numbers of five significant places. The writer has already called such an accuracy fictitious, as probably Mr. Bull will agree. However, provided the technique in obtaining elastic curves is properly under control, the results would be more reliable—that is, philosophically more sound—than results obtained by conventional mathematical analyses such as Mr. Bull and the writer have used.

The so-called exact analyses used in checks Nos. 1 to 6 were based on the conventional assumption that the principle of superposition applies; namely, that the rings maintain their original shape even in an analysis made for the purpose of obtaining changes in shape. Consider that the model from which the elastic curve is obtained is not perfectly elastic and thus violates elastic theory. How about applying an elastic theory to a structure as imperfectly elastic as reinforced concrete? In such a case, is the structure wrong or is the theory wrong? The philosophy of elastic curves as influence lines is sounder than any other mathematical theory of elasticity known to the writer because it is not restricted by assumptions of superposition and perfect elasticity as other theories of elasticity commonly are. Even the extremely small deformations, which often are stipulated, are philosophically not essential.

To some engineers who are not overly familiar with the science of soil mechanics, it may appear difficult to realize how tangential forces may be transmitted from earth to a smooth steel shell. However, in the shells of the aeroplane fuselages the shear forces are very real and are all directed tangentially. This leads the writer to surmise that Mr. Bull's carefully composed tables may well find wider application in other fields than merely the field of tunnel stresses for which they were originally computed.

The writer takes pleasure in expressing his admiration for, and his sense of indebtedness to, Mr. Bull's contribution. He expects to see Mr. Bull's theory and methods applied to a variety of problems, some of them possibly of greater importance, economically, than linings of shield-driven tunnels.

Corrections for *Transactions:* In November, 1944, *Proceedings*, page 1386, change the third line of thrust at Joint F from "− 9.778" to "+ 9.778"; and, on page 1391, line 32, change "9/16 in." to "9/32 in."

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DISCUSSIONS

TRANSLATORY WAVES IN NATURAL CHANNELS

Discussion

BY HAROLD A. WEGGEL, AND W. M. LANSFORD

HAROLD A. WEGGEL,[4] JUN. AM. SOC. C. E.[4a]—A noteworthy detailed study of translatory waves is presented in this paper; yet in a positive sense the conclusions enumerated do not seem to compensate for the effort expended. It would be desirable to go one step further than simply to prove or disprove the validity of the use of certain theoretical formulas. Some of the conclusions have been known since 1842–1843 when John Scott Russell presented the "Report on Waves"[5] to the British Association for the Advancement of Science; and later experimenters have observed the remaining conclusions. However, very few observers have been able to apply corrections for change in slope of water surface, friction, and channel storage. The data in the paper seem to indicate that the channel characteristics could be determined for the particular waves studied so that corrections could be applied to the theoretical formulas.

The velocity of the wave is affected by the slope of the original water surface, being greater for steep channels and less for channels of flat slope. In the limiting condition of steep slope, the velocity of a small wave is twice the mean velocity of the channel, and, in the other extreme of a flat slope, the velocity of the wave is equal to the surface velocity of the channel.[6,7]

Theoretically, the wave velocity in quiet water is $\sqrt{g\,y_m}$ but experiments by A. Zeitlinger[8] at the Hydraulic Laboratory in Graz, Austria, showed that, in shallow water, friction caused a decrease in the velocity of the wave to

$$c = 0.91\sqrt{g\,y_m} \qquad (3)$$

NOTE.—This paper by J. H. Wilkinson, M. Am. Soc. C. E., was published in June, 1944, *Proceedings*.

[4] Associate Structural and Hydr. Engr., Havens and Emerson, Cleveland, Ohio.

[4a] Received by the Secretary October 19, 1944.

[5] "Report on Waves," by J. Scott Russell, *Report of the Fourteenth Meeting of the British Association for the Advancement of Science*, London, 1844, p. 311.

[6] "A Study in Flood Waves," by Elmer E. Moots, thesis presented to the State University of Iowa in Iowa City in February, 1927, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[7] Letter to the editor of *Civil Engineering* from L. K. Sherman, discussing a paper, "Translatory Waves in Open Channels," by H. W. King, *Civil Engineering*, August, 1933, p. 473.

[8] "Dam-Break Waves," by Armin Schoklitsch, Mathematisch-Naturwissenschaftliche Classe, Vol. 126, 2A, 1917, pp. 1489–1514.

Perhaps a correction factor similar to Eq. 3 would make the results of Eq. 2*b* comparable with, instead of consistently higher than, observed velocities.

In a similar manner, Eq. 1*b* should be changed to

$$c_2 = V_1 + \frac{(V_2 - V_1)\, A_2}{A_2 - A_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Eq. 4 separates the superimposed wave from the initial velocity V_1 so that a correction factor for wave friction may be applied. It is apparent from Figs. 4(*b*) and 4(*d*) that this friction factor must be relatively constant over a wide range for the midpoint-of-rise and for the midpoint-of-fall. In the solitary wave of translation these points are the only parts of the wave that really travel as a wave front or movement of constant flow or wave height.

In the case of translatory waves on rivers of flat slope, such as the lower section of the Tennessee River, it is realized that some of the difficulties of the problem are practically insurmountable due to channel storage. Nevertheless, it may be possible to obtain some empirical coefficient which would give closer agreement between formulas and observed data on translatory waves.

Perhaps a study of the reflected waves in the Decatur section (see Fig. 5) may be helpful in an approximate determination of a factor to be applied to the velocity of the channel, since a difference of 5 miles per hr was noted for the wave travel (see heading, "Translatory Waves in Wheeler Reservoir: Velocities of Translatory Waves in Wheeler Reservoir"). The time of surges in Wheeler Reservoir, about 11 hr over a distance of 2 × 74 miles, or 148 miles, gives 13.5 miles per hr, or the average velocity downstream only. Do the surges travel upstream at the same rate or at some rate differing by a factor of channel velocity, such as that indicated by the equation:[9,10]

$$c = K\sqrt{g\, y_m} \pm (\tfrac{1}{4}\, V_1 + \tfrac{3}{4}\, V_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

A solution of these perplexing problems would do much to enhance the knowledge of the variations in velocities of translatory waves.

W. M. LANSFORD,[11] M. AM. SOC. C. E.[11a]—The problem of translatory waves is of much importance to hydraulic engineers, and the author has presented interesting and valuable data on the subject.

A few years ago the writer conducted some experiments on translatory waves in a rectangular channel in the Hydraulics Laboratory at the University of Illinois at Urbana. The channel used was made of concrete 5 ft wide and 4.5 ft deep and had a straight horizontal length between head gate and tail gate of 147 ft. The tail gate was used to control the depth and the head gate was used to help control the discharge. A vertical steel plate ½ in. thick which fitted into grooves cut in the sides of the channel was dropped across the channel to stop the flow, thereby causing a wave to travel upstream. This gate was placed 7 ft upstream from the tail gate.

[9] "Wave Movements," by Hubert Engels, in "Handbuch des Wasserbaues," I Band, W. Engelmann, Leipzig and Berlin, 1914, pp. 125–138.

[10] "Traveling Waves," by Alexander Koch and Max Carstanjen, in "Bewegung des Wassers," Julius Springer, Berlin, 1926, pp. 132–150.

[11] Associate Prof., Theoretical and Applied Mechanics, Talbot Laboratory, Univ. of Illinois, Urbana, Ill.

[11a] Received by the Secretary November 1, 1944.

The velocity of the wave was measured over a 114.6-ft length of the channel by means of floats connected by electric contacts to a pen which recorded on a revolving drum. The water used was taken from a standpipe from which approximately 6,500 gal per min could be supplied at a constant head of about 65 ft.

TABLE 10.—TRANSLATORY WAVES IN A RECTANGULAR CHANNEL

A.—Discharge Constant					*B.*—Initial Velocity Constant					*C.*—Depth Constant				
Test No.	Depth $\bar{y}_1$ (ft)	Velocities (Ft Per Sec)			Test No.	Discharge (cu ft per sec)	Depth $\bar{y}_1$ (ft)	Wave Velocities (Ft Per Sec)		Test No.	Discharge (cu ft per sec)	Velocities (Ft Per Sec)		
		Flow (initial)	Wave									Flow (initial)	Wave	
			Observed	Computed				Observed	Computed				Observed	Computed
(*a*) Q = 3.5 Cu Ft Per Sec					(*a*) V_1 = 0.50 Ft Per Sec					(*a*) $\bar{y}_1$ = 2.10 Ft				
71	1.2	0.58	6.19	6.13	40	3.5	1.40	6.22	6.66	81	3.00	0.29	8.02	8.20
72	1.4	0.50	6.55	6.66	41	4.0	1.60	6.93	7.01	82	3.50	0.34	8.13	8.20
73	1.8	0.39	7.14	7.59	42	4.5	1.80	7.40	7.52	83	4.00	0.38	7.90	8.18
74	1.7	0.40	7.55	7.30	43	5.0	2.00	7.52	7.88	84	4.50	0.43	7.90	8.15
75	2.0	0.35	7.78	7.96	44	5.5	2.20	8.40	8.35	85	5.0	0.48	7.75	8.11
76	2.2	0.32	8.12	8.33	45	6.0	2.40	8.79	8.69	86	5.5	0.52	7.87	8.13
77	2.3	0.30	8.25	8.56	46	6.5	2.60	8.76	9.04	87	6.0	0.57	8.03	8.10
78	2.4	0.28	8.61	8.73	47	7.0	2.80	9.14	9.40	88	6.5	0.62	8.07	8.17
79	2.6	0.27	9.15	9.00	48	7.25	2.90	9.50	9.55	....	....	....	....	....
80	2.8	0.25	9.00	9.45	49	7.5	3.00	9.43	9.71	....	....	....	....	....
(*b*) Q = 5.0 Cu Ft Per Sec					(*b*) V_1 = 0.60 Ft Per Sec					(*b*) $\bar{y}_1$ = 1.50 Ft[a]				
50	1.40	0.71	6.68	6.74	30	6.88	1.28	6.32	6.42	11	3.83	0.51	6.86	6.84
51	1.60	0.63	7.11	7.08	31	4.90	1.63	7.05	7.19	12	4.48	0.60	6.83	6.79
52	1.80	0.56	7.43	7.43	32	5.42	1.81	7.37	7.61	13	4.90	0.65	6.72	6.85
53	2.0	0.50	7.80	7.89	33	6.1	2.03	7.85	7.97	14	5.42	0.71[a]	6.78	6.79
54	2.2	0.45	8.29	8.29	34	6.6	2.20	8.32	8.37	15	6.10	0.80[a]	6.78	6.74
55	2.4	0.42	8.53	8.73	35	7.2	2.40	8.44	8.62	16	6.55	0.87[a]	6.73	6.81
56	2.5	0.40	8.95	8.91	36	7.8	2.60	8.92	8.98	17	6.77	1.11	6.70	6.65
57	2.6	0.39	8.81	9.08	37	8.4	2.80	9.13	9.34	18	7.55	1.01	6.70	6.75
58	2.7	0.37	9.37	9.29	38	8.7	2.90	9.43	9.65	19	8.10	1.08	6.64	6.72
59	2.9	0.35	9.55	9.54	39	9.0	3.00	9.43	9.70	20	8.60	1.15	6.77	6.72
60	3.0	0.33	9.55	9.81	....	....	....	....	....	....	....	....	....	....
(*c*) Q = 7.25 Cu Ft Per Sec					(*c*) V_1 = 0.70 Ft Per Sec					(*c*) $\bar{y}_1$ = 1.30 Ft				
3	1.07	1.36	6.04	6.56	61	4.2	1.20	5.94	6.22	21	3.50	0.54	6.32	6.23
4	1.39	1.04	6.74	6.65	62	4.9	1.40	6.62	6.60	22	4.53	0.70	6.40	6.40
5	1.76	0.82	7.62	7.57	63	5.3	1.50	7.10	6.80	23	5.30	0.81	6.31	6.31
1	2.06	0.70	8.14	8.14	64	5.6	1.6	6.92	6.99	24	5.81	0.90	6.33	6.36
6	2.19	0.66	8.22	8.27	65	5.95	1.7	7.20	7.20	25	6.18	0.95	6.33	6.34
7	2.42	0.60	8.75	8.85	66	6.3	1.8	7.52	7.51	26	6.90	1.06	6.32	6.31
8	2.59	0.56	9.05	9.14	67	7.0	2.0	8.02	7.92	27	7.30	1.12	6.29	6.26
9	2.91	0.50	9.46	9.65	68	7.7	2.2	8.03	8.21	28	7.70	1.19	6.26	6.30
10	3.07	0.47	10.32	9.93	69	8.4	2.4	8.54	8.60	29	8.10	1.25	6.20	6.23
....	....	....	....	....	70	9.1	2.6	8.65	8.94	....	....	....	....	....

[a] Exceptions: Test No. 14, $\bar{y}_1$ = 1.52; Test No. 15, $\bar{y}_1$ = 1.52; and Test No. 16, $\bar{y}_1$ = 1.51.

The observed wave velocities were compared with the theoretical values as computed from the formula proposed by Professor King[12,13] for abrupt waves in

[12] "Translatory Waves in Open Channels," by H. W. King, *Civil Engineering*, June, 1933, Eq. 8, p. 320.
[13] "Handbook of Hydraulics," by H. W. King, 3d Ed., 1939, pp. 421–422.

open channels. Professor King's formula is

$$c_w = \pm \sqrt{\frac{g\, y_1}{2\, y_2}(y_2 + y_1)} + V_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which c_w is the velocity of the wave, in feet per second; V_1 is the initial velocity of the water, in feet per second; and y_1 and y_2 are the depths in feet before and after the wave, respectively. The results of these tests are given in Table 10.

The results given in Table 10*A* are for tests in which the discharge was maintained at a constant value. In Table 10*B* the initial velocity was kept constant, and in Table 10*C* the depth y_1 was kept constant. In all the tests the percentage difference between observed and computed values of the wave velocities is small except in extreme cases of very low or very high values. The effect of the depth y_1 seems more pronounced than it should because of the limited range over which these tests were made.

Although the data presented by the author in Table 9 show a rather wide variation between observed values of wave velocity and values computed by use of Eq. 2*a*, the data in Table 10 seem to indicate a fairly good agreement between these values. As mentioned by the author, the widening of the stream very likely accounts for some of the discrepancies. Within the limits of the tests herein presented Eq. 6 seems to yield reliable results.

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DISCUSSIONS

WEDGE-SHAPED STRUCTURAL MEMBERS UNDER DIRECT STRESS AND BENDING

Discussion

BY GEORGE WINTER

GEORGE WINTER,[13] M. AM. SOC. C. E.[13a]—The ingenuous analysis of an important problem, as presented in this paper, is based on the assumption that plane sections remain plane in the deformed or stressed state of the members. This assumption is rigorously true for beams in pure bending. For transversely loaded beams it can be shown that the error introduced by such an assumption is insignificant provided the span : depth ratio is not too small. For beams of span : depth ratio smaller than about 3, such as occur occasionally in bunkers when the vertical walls are designed as beams spanning between supporting columns, the assumption of a straight-line strain distribution leads to very substantial errors.[14] This is true despite the fact that, in such beams, edges are parallel.

In the case of a wedge-shaped member it appears pertinent to question whether the assumption of a straight-line strain distribution can be expected to give reasonably accurate results, for the following two reasons: (*a*) In such members, except for very small wedge angles, the ratio of length of element to average depth of cross section is rather small—by analogy with the aforementioned prismatic beams, one may then expect a nonlinear strain distribution for essentially identical reasons; and (*b*) in addition, the sloped shape of the member is likely to introduce further deviations from the familiar situation in long, shallow prismatic beams.

It is customary to assume that the strain distribution in a reinforced concrete member is the same as that in an analogous member made of homogeneous material. This enables one to apply the results of the theory of elasticity of such homogeneous members without substantial change to the design of rein-

NOTE.—This paper by Robert B. B. Moorman, Assoc. M. Am. Soc. C. E., was published in June, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1944, by Phil M. Ferguson, and Stanley U. Benscoter.

[13] Asst. Prof., Civ. Eng., Cornell Univ., Ithaca, N. Y.

[13a] Received by the Secretary December 20, 1944.

[14] "Über den Spannungszustand in hohen Trägern und die Bewehrung von Eisenbetontragwänden," by Hermann Bay, Berlin, 1931.

forced concrete structures. Although it is likely that this assumption is not strictly correct, it has proved its worth in practical application. Proceeding on this assumption, it is necessary, in the present case, to investigate the strain distribution in a homogeneous, wedge-shaped member under bending plus thrust. The fundamental theory pertinent to this problem has been given by S. Timoshenko,[15] who shows that, in a wedge loaded as in Fig. 9, the principal

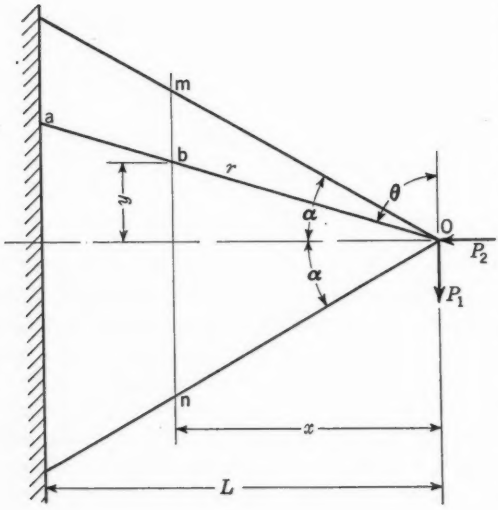

FIG. 9

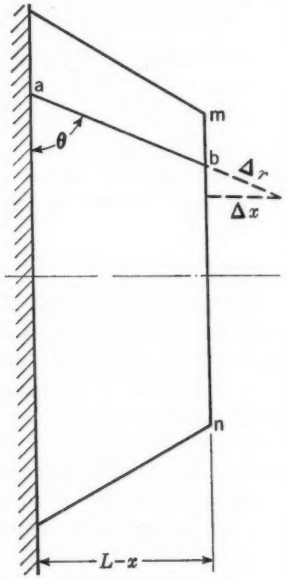

FIG. 10

tension and compression stresses are directed radially. The author's case of bending plus thrust can be resolved into the two individual cases of transverse bending by a force P_1 in Fig. 9 and of longitudinal thrust by a force P_2. The stress-strain distribution due to simultaneous action of P_1 and P_2 is then obtained by simple superposition.

The strain distribution in bending due to P_1 will first be investigated. The magnitude of the radial stress at point b (Fig. 9), distance r from point O on a radius subtending the angle θ with the direction of the force P_1,[16] is

$$f_r = \gamma_1 \frac{\cos \theta}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

in which

$$\gamma_1 = -\frac{P_1}{\alpha - 0.5 \sin (2\alpha)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

is a constant depending on the wedge angle and the magnitude of the load P_1.

[15] "Theory of Elasticity," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1934, pp. 93–95.

[16] *Ibid.*, Eq. 70, p. 94.

Consequently, the radial strain is

$$\epsilon_r = \frac{\gamma_1 \cos\theta}{E} \frac{}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

To obtain the shape of an originally plane cross section mn (Fig. 9), it is necessary to calculate the change of length Δ_{ab} of a fiber of undistorted length ab (Fig. 10). This change of length is obtained by integrating the radial strain (Eq. 40) over the length of fiber ab. Thus,

$$\Delta_{ab} = \frac{\gamma_1 \cos\theta}{E} \int_{r_b}^{r_a} \frac{dr}{r} = \frac{\gamma_1 \cos\theta}{E} \log_e \frac{r_a}{r_b} \quad \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

in which r_a and r_b are, respectively, the radial lengths Oa and Ob (Fig. 9). From the angular relations of Fig. 9,

$$r_a = \frac{L}{\sin\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42a)$$

and

$$r_b = \frac{x}{\sin\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42b)$$

and, by substitution of these values in Eq. 41,

$$\Delta_{ab} = \frac{\gamma_1 \cos\theta}{E} \log_e \frac{L}{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

The horizontal component of the displacement Δ_{ab} of point b is

$$\Delta x = \Delta_{ab} \sin\theta = \frac{\gamma_1}{2E} \left[\log_e \frac{L}{x} \right] \sin(2\theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

On the other hand, the magnitude of the distance from the axis, y (Fig. 9), is

$$y = x \cot\theta = x \frac{\sin(2\theta)}{1 - \cos(2\theta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

A comparison of Eqs. 44 and 45 shows that, whereas y (the distance from the axis) is proportional to $\frac{\sin(2\theta)}{1 - \cos(2\theta)}$, the magnitude of the horizontal displacement Δx is proportional to $\sin(2\theta)$. It is thus established that, for the case of transverse bending, strains are not proportional to y in wedge-shaped members and that, consequently, originally plane cross sections become curved under such loading.

The shape of a cross section of a member subject to axial thrust is easily investigated in the same manner. With the angle θ as defined in Fig. 9, the radial stress in this case[17] is

$$f_r = \gamma_2 \frac{\sin\theta}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

[17] "Theory of Elasticity," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1934, Eq. 69, p. 94.

in which

$$\gamma_2 = -\frac{P_2}{\alpha + 0.5 \sin (2\alpha)} \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

The horizontal component of the displacement, obtained by the same reasoning as before, is, in this case,

$$\Delta x = \frac{\gamma_1}{2E}\left[\log_e \frac{L}{x}\right] \sin^2 \theta \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

This displacement is not constant over the cross section, but is proportional to $\sin^2 \theta$. Thus, as in the case of transverse bending, plane cross sections become curved in a wedge-shaped member subject to radial thrust.

To facilitate visualizing the magnitude of this effect, displacements were computed from Eqs. 43 and 48 for the case of a wedge with $\alpha = 30°$. The shapes of the distorted cross sections are plotted to scale—in Fig. 11(*a*) for

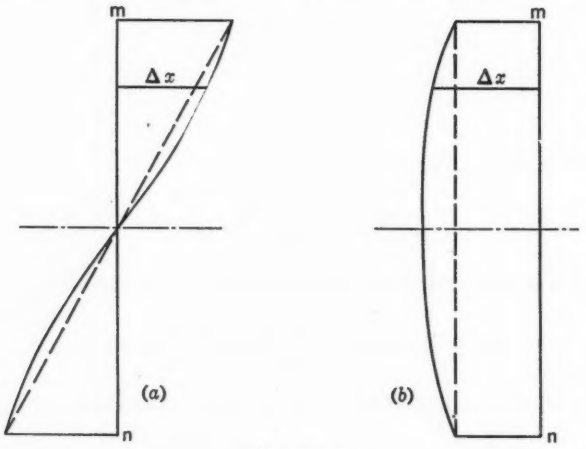

FIG. 11

transverse bending and in Fig. 11(*b*) for radial thrust. Even for this relatively small angle α, the deviations from planeness are substantial. For example, in the case of axial thrust, with $\alpha = 30°$, the horizontal displacement along the axis is one third larger than that at the outer fibers, whereas, if plane cross sections were maintained, these two displacements would have to be the same. Figs. 11(*a*) and 11(*b*) further show that in the case of simultaneous bending plus thrust, by superposition, the curvatures of the section subtract on the tension side, but add on the compression side. This complicates further the final shape of the cross section, which will also depend on the ratio $P_1 : P_2$.

An exact evaluation of the numerical influence of this distortion on the design values computed on the assumption of plane deformation is far beyond the scope of these sketchy comments. The numerical magnitude of the aforementioned discrepancies suggests, however, that practical application of the author's findings should probably be delayed until a more rigorous investigation of this problem becomes available.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

RAINBOW ARCH BRIDGE OVER NIAGARA GORGE

A SYMPOSIUM

Discussion

BY SHORTRIDGE HARDESTY, J. M. GARRELTS, AND I. G. HEDRICK, JR.; ELMER K. TIMBY, LAWRENCE M. MEAD, JR., AND ROBERT MCELDOWNEY, JR.; AND JOHN I. COPP, KARL DE VRIES, WILLIAM H. JAMESON, AND JONATHAN JONES

SHORTRIDGE HARDESTY[60] AND J. M. GARRELTS,[61] MEMBERS, AM. SOC. C. E., AND I. G. HEDRICK, JR.,[62] ASSOC. M. AM. SOC. C. E.[62a]—The criticisms and extensions in the discussions have materially increased the usefulness of this Symposium and are appreciated by the writers.

Several discussers have objected to the use of the term, "elastic theory," to denote a method of structural analysis based on the assumption that the change in the geometry of the arch axis under load does not affect the stresses in the rib, and to the use of the term, "deflection theory," to denote a method which takes into account the effect of such change in geometry and thus gives a more nearly correct solution than does the elastic theory. To the writers the use of two terms seems merely to be carrying into arch analysis the nomenclature long used in the field of suspension bridges. The two usages[63,64] are exactly parallel.

NOTE.—This Symposium was published in October, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1943, by Egidio O. Di Genova, and Charles Mackintosh; January, 1944, by C. M. Goodrich, and T. Kennard Thomson; February, 1944, by C. M. Spofford, L. J. Mensch, and O. H. Ammann; March, 1944, by Louis Balog, William G. Rapp, and Leon Beskin; April, 1944, by Neil Van Eenam, and C. H. Gronquist; May, 1944, by M. Hirschthal; and June, 1944, by George Stern.

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[61] Associate Prof., Civ. Eng., Columbia Univ.; Designing Engr., Waddell & Hardesty, New York, N. Y.

[62] Stress Analyst, Grumman Aircraft Eng. Corp., Bethpage, L. I., N. Y.

[62a] Received by the Secretary December 18, 1944.

[63] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1929, p. 246.

[64] "Elementary Treatise on Statically Indeterminate Stresses," by John I. Parcel and G. A. Maney, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1936, p. 351.

The writers note that some discussers have apparently confused the loading used in the design of the arch rib with the half-span loading considered in Part II of the paper. In analyzing the stresses in the arch ribs, the moments and thrusts at each section were computed for a unit load at each panel point in turn. The live-load stresses at each section were then determined by loading those parts of the span that would result in the maximum stresses. The use of a single arbitrary length of loading for the design of the entire rib would give stresses much less than the maximum stress for most sections; hence such method would be seriously in error.

Mr. Di Genova's discussion, in general, is irrelevant to the subject of the paper. The answer to his claims of exactness for the "elastic theory" can be found by a careful reading of the paper, in connection with discussions on it, and also by reference to any good treatise on the "deflection theory" for suspension bridges. The deflections of the 950-ft two-hinged arch, which Mr. Di Genova says are relatively small, actually have the effect of increasing the bending moment at the quarter point by 72% and of decreasing the safety factor of the rib from 1.875 to 1.30.

Mr. Mackintosh's suggestion of combining M_{DL} with M_{LL} would give design moments which are too large unless the dead loading was unsymmetrical and produced moments of nearly the same spanwise variation as the unsymmetrical live loading of the critical condition. Ordinarily this is not the case, as dead-load moments and deflections are usually symmetrical, whereas the rib has the lowest resistance to buckling with a two-loop bending-moment diagram having positive moment over approximately one half the span and negative moment over the remaining part of the span.

Mr. Mackintosh, and later Mr. Beskin, have suggested an approximate beam-column correction of the form:

$$M_f = M_{ae}\left[\frac{1}{1-\frac{T}{T_{cr}}}\right] \qquad (116)$$

The writers agree that this correction is sufficient when the rib is of uniform or nearly uniform section and when T is nearly constant throughout the length of the rib; but they were not willing to depend entirely on this form of approximation for the Niagara arch, where considerable variation existed in cross section and curvature of the rib axis. An evaluation of the approximations involved in this form of correction will entail essentially the computations referred to by the writers as their "exact" procedure.

Mr. Goodrich's request for an explanation of "Stern's Theory" has been supplied in a later discussion of the present paper by Mr. Stern.

Professor Spofford has suggested that a cost and quantity estimate would be a worth-while addition to the paper. The writers, therefore, have included the following tabulation of the final costs (1940–1941 exchange values):

Detailed Construction Costs:

Contract 1A. Substructure on the United States Side (McLain Construction Corporation)—		
Unclassified excavation (4,735 cu yd at $5)	$ 23,675	
Concrete in footings (2,559 cu yd at $10)	25,590	
Concrete in arch abutments above footings (1,095 cu yd at $11)	12,045	
Concrete in shafts (3,061 cu yd at $8.50)	26,018	
Bar reinforcement for structures (369,039 lb at 4¢)	14,762	
Setting anchor bolts	1,000	
Core borings (60 lin ft at $4)	240	
Total for contract 1A, United States money		$ 103,330
Contract 2A. Substructure on Canadian Side (Aiken and MacLachlan)—		
Unclassified excavation (6,117 cu yd at $2.35)	$ 14,374	
Concrete in footings (3,224 cu yd at $12)	38,688	
Concrete in arch abutments above footings (1,780 cu yd at $12.50)	22,250	
Concrete in shafts (3,236 cu yd at $12.75)	41,259	
Bar reinforcement for structures (418,453 lb at 4½¢)	18,830	
Setting anchor bolts	640	
Core borings (105 lin ft at $5)	525	
Total for contract 2A { Canadian money	$ 136,566	
Total for contract 2A { United States money		$ 120,246
Contract 3A. Superstructure of Arch Span (Bethlehem Steel Company)—		
Concrete in arch abutments (390 cu yd at $33.50)	$ 13,065	
Concrete in slabs and curbs (1,405 cu yd at $27)	37,935	
Bar reinforcement for structures (209,264 lb at 4.47¢)	9,354	
Reinforcing trusses for roadway slabs (305,136 lb at 5.75¢)	17,545	
Carbon steel in floor system 2,450 lb × 983 ft (2,407,527 lb at 8.232¢)	198,188	
Silicon steel in floor system 495 lb × 983 ft (485,594 lb at 8.9¢)	43,218	
Carbon steel in spandrel columns 1,195 lb × 950 ft (1,136,301 lb at 8.694¢)	98,790	
Carbon steel in rib laterals 950 lb × 950 ft (903,296 lb at 13.848¢)	125,088	
Silicon steel arch ribs and shoes 5,455 lb × 950 ft + 4 × 260,000 (6,223,496 lb at 14.525¢)	903,963	
Provision for utilities	2,823	
Extra work orders	1,812	
Total for contract 3A, United States money		$1,451,781

Contract 4. Superstructure of American Approach Spans (McLain Construction Corporation)—		
Unclassified excavation (1,000 cu yd at $1.50)	$ 1,500	
Concrete in footings (310 cu yd at $11)	3,410	
Concrete in abutments above footing (216 cu yd at $16)	3,456	
Concrete in superstructure (3,368 cu yd at $27)	90,963	
Bar reinforcement for structures (593,303 lb at 4¢)	23,732	
Structural steel (20,741 lb at 15¢)	3,111	
Provision for utilities	500	
Earth excavation—special (5,000 cu yd at 45¢)	2,250	
Rock excavation—special (204 cu yd at $2)	408	
Protection of cliff	3,867	
Total for contract 4, United States money		$ 133,197
Contract 5. Superstructure of Canadian Approach Spans (Aiken and MacLachlan)—		
Unclassified excavation (1,299 cu yd at $2.50)	$ 3,247	
Concrete in footings (864 cu yd at $13)	11,232	
Concrete in shafts (1,188 cu yd at $16)	19,008	
Concrete in superstructure of river road span (1,118 cu yd at $16)	17,888	
Concrete in superstructure of arch spans (3,334 cu yd at $27)	90,018	
Bar reinforcement for structures (1,020,300 lb at 6¢)	61,218	
Structural steel (63,018 lb at 30¢)	18,905	
Masonry facing (2,500 cu ft at $6.50)	16,250	
Arch stones (900 cu ft at $8)	7,200	
Special stones (2,600 cu ft at $10)	20,000	
Special stone details	1,550	
Provision for utilities	3,000	
Extra work orders	750	
Sales taxes	2,924	
Total for contract 5 {Canadian money	$ 273,190	
Total for contract 5 {United States money		$ 241,783
Contract AP1. Retaining Walls at American End (Charles H. Wing Company)—		
Unclassified excavation for wall footings (3,141 cu yd at $1.25)	$ 3,927	
Unclassified excavation for parkway drive (9,063 cu yd at 54¢)	4,894	
Concrete in wall footings (499 cu yd at $11.15)	5,565	
Concrete in walls above footings (555 cu yd at $21.75)	12,072	
Concrete in railings (75 cu yd at $26.56)	1,992	
Bar reinforcement for structures (84,191 lb at 3.76¢)	3,166	
Extra work orders	2,300	
Total for contract AP1, United States money		$ 33,916

Summary of Construction Costs:

Bridge Between Abutments—		
Contract 1A, substructure on American side	$ 103,330	
Contract 2A, substructure on Canadian side	120,246	
Contract 3A, superstructure of arch span	1,451,781	
Contract 4, superstructure of American approach	133,197	
Contract 5, superstructure of Canadian approach	241,783	
Retaining walls and excavation—American side	33,916	
Lighting system on bridge	12,973	
Total bridge between abutments		$2,097,226
American Terminal—		
Paving and embankment of plaza	$ 63,743	
Paving and embankment of access road	50,519	
Buildings	166,000	
Lighting system	13,000	
Total American terminal		$ 293,262
Canadian terminal, cost to bridge commission		160,000
Name plates		2,000
Toll equipment		10,820
Office equipment and fixtures		16,000
Miscellaneous construction items		11,100
Borings		7,925
Engineering, architectural, and inspection services		193,989
Total construction cost (United States money)		$2,792,322

Mr. Mensch states that "J. A. C. Bresse warned his students not to use ribs so shallow as to cause excessive deflection due to a moving load. No real, valid reason has been given in this Symposium why this sound warning has been disregarded." Deflections were not disregarded in any sense. In fact, Part II of the paper is devoted entirely to the discussion of methods for evaluating the conditions which determine excessive deflections. The deflections of the final design are within permissible limits. Eqs. 18 and 19 are correct, and Eqs. 20 and 21 are clearly stated to be approximations. Mr. Mensch's Eqs. 39 to 51 are perhaps satisfactory for obtaining approximate sections. These equations apply the "elastic theory" to a particular case, the parabolic rib with $I = I_c \sec \alpha$, which yields simple integrals.

Mr. Mensch also appraises the paper erroneously when he states: "Contrary to the implied limitations expressed by the Symposium authors, Eqs. 39 to 51 are quite sufficient for the design of an arch of the Rainbow type, and the results agree well with those given in Table 1." Table 1 contains "elastic theory" results which the writers obtained by a more accurate integration, for this particular arch, than would be given by Mr. Mensch's equations. The comparison, therefore, has no bearing on the sufficiency of the "elastic theory." However, the approximations of Mr. Mensch's equations are hardly negligible even if deflection effects are neglected. His value of H, computed as 5,810 kips instead of the more accurate value of 5,911 kips, would represent a difference in moment at points near the crown of 101 kips × 150 ft, or of approxi-

mately 15,000 ft-kips. In comparing deflections, Mr. Mensch has added deflections at the crown point due to dead load, live load, rib shortening, and temperature to obtain a total of 14.48 in., which he has compared to the writers' value of 6.9 in. for (dead + live + impact)-load at the quarter point. This comparison has no meaning since the deflections are for different points of the arch rib and for different loads. The true comparison was given in the paper as 5.5 in. for the elastic theory as compared to 6.9 in. for the correct computations.

Under the heading, "Introduction," in the third paper of the Symposium it is stated that the arch was cambered for full dead load.

Mr. Ammann's contention, that the permissible limits for stresses should be assigned to the accurate stresses and not to those which are approximate, is strictly correct. As a matter of fact the designers applied this test in checking the most critical sections, but did not follow this procedure in preparing the stress sheet of the drawings because of the practical difficulties involved in the computation and preparation of a complete stress sheet showing the deflection stresses at every panel point.

The writers wish to thank Mr. Balog for calling attention to the Dischinger method of calculating deflection effects with its numerical application to the Rainbow arch rib.

Mr. Beskin has objected to the use of the word "theories" by the writers in referring to certain studies (see Appendix I) which have been made in connection with the deflection effects of arches. Since the writers were employing the papers in question as references, they accepted the titles of the original authors, who, correctly or incorrectly, used the term "theory" in describing their work.

Mr. Beskin has rightfully questioned the use of safety factors as they appear in Eq. 16. Although it is generally true that allowance should be made in design for increases in actual dead loads and live loads, in most structures the maximum temperature and wind loads are constant throughout the life of the structure. The factor of safety for these loadings, therefore, need serve only as a factor of ignorance to cover estimation of the present loads and approximations of the analysis. A factor of $N' = 1.33$ used with the 30-lb wind loading of present specifications and the maximum temperature range of the locality would seem to be sufficient.

The effect of axial thrust, although finally neglected, is not correctly expressed in Eq. 2. A more complete equation is:

$$\frac{d^2u}{ds^2} + \frac{u}{(r_o)^2} = -\frac{M}{E\,I} - \frac{T\,\mu}{r_o\,A\,E} + \left(\frac{M}{E\,I} + \frac{T}{r_o\,A\,E} + \frac{T\,\mu}{r_o\,A\,E}\right)\frac{T}{A\,E} + \frac{1}{2\,r_o}\left(\int\frac{T\,ds}{r_o\,A\,E} + \int\frac{M}{E\,I}\,ds + \int\frac{T\,\mu}{r_o\,A\,E}\,ds + C\right)^2 \quad \ldots\ldots (117)$$

in which μ = Poisson's ratio.

To illustrate the effect of axial deformation on radial deflections, consider the semicircular arch in Fig. 44 with radial loads of magnitude T. The bending

moment in this case is zero throughout and the axial compression T is constant around the full length of the rib. It will be assumed that $\mu = 0$. If point A is assumed fixed and point B is free to deflect, then the tangential component of deflection, v, of point B is seen to be

$$v = \frac{\pi R T}{A E} \quad \ldots\ldots\ldots\ldots (118)$$

and the radial component of deflection of point B is

$$u = \frac{v^2}{2R} = \frac{R}{2}\left(\frac{\pi T}{A E}\right)^2 \quad \ldots\ldots (119)$$

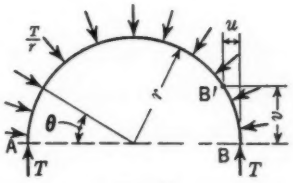

FIG. 44

The same result can be obtained by Eq. 117, which reduces to the following when $M = 0$,

$$\frac{d^2u}{ds^2} + \frac{u}{R^2} = -\frac{1}{R}\left(\frac{T}{A E}\right)^2 + \frac{1}{2R^3}\left(\int \frac{T}{A E}\, ds\right)^2 \quad \ldots\ldots\ldots (120)$$

The solution of Eq. 120 for constant T is

$$u = \frac{1}{2R}\left(\int \frac{T}{A E}\, ds\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (121)$$

and, since $ds = R\, d\theta$,

$$u = \frac{R}{2}\left[\left(\frac{T\theta}{A E}\right)^2\right]_0^{\pi} = \frac{R}{2}\left(\frac{T\pi}{A E}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots (122)$$

This question of the effect of thrust on radial deflections, however, is academic, since both Mr. Beskin and the writers agree that its influence is negligible for steel or concrete ribs.

The writers inadvertently made the statement in the paragraph containing Eq. 6 that the term $\frac{u}{(r_o)^2}$ of Eq. 2 is neglected in the elastic theory and, therefore, in their "exact" procedure. The basic differential relation[65] of the "elastic" theory is

$$\frac{\Delta\, d\theta}{ds} = -\frac{M}{E I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (123)$$

in which $\Delta\, d\theta$ is the change in angle between the end tangents of the element ds. In the paper it was stated incorrectly that $\frac{d\theta}{ds} = -\frac{d^2u}{ds^2}$—a relation that is exact for straight beams only. It can be proved that for curved beams the following relations[66] exist:

$$\frac{\Delta\, d\theta}{ds} = \left(\frac{1}{r_f} - \frac{1}{r_o}\right) = \frac{d^2u}{ds^2} + \frac{u}{(r_o)^2} = -\frac{M}{E I} \quad \ldots\ldots\ldots (124)$$

in which the element ds is assumed to remain of constant length.

[65] "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, 10th Ed., John Wiley & Sons, Inc., New York, N. Y., 1929, Pt. II.

[66] "Strength of Materials," by Stephen Timoshenko, 1st Ed., D. Van Nostrand Co., Inc., New York, N. Y., 1940, Pt. II, p. 457.

The actual working equation used in obtaining deflections by the elastic theory was:

$$\delta_a = \int_A^B \frac{M_{ae}\, m}{E\, I}\, ds \qquad (125)$$

in which m is the moment in the arch due to a unit load at point a in the direction of the deflection δ_a. Since both M_{ae} and m include the effect of $\frac{u}{(r_o)^2}$, the deflections and moments of both the "elastic" analysis and the "exact" procedure have been computed without neglect of the term $\frac{u}{(r_o)^2}$.

With regard to the use of an average value of T in Eq. 16 instead of the thrust at the point A which is being analyzed, the thrust near the quarter point is, in the first place, very nearly the average thrust. Secondly, since the change in H is negligible for deflection effects, the position of the thrust line remains very close to the position obtained in the elastic analysis, which accounts for the relation $\Delta m_{a1} = T_a\, u_{ae}$. Mr. Beskin is correct in stating that the thrust in the denominator should include the thrust due to wind and temperature.

Eq. 16 for the two-hinged rib, when revised in accordance with the foregoing discussion, becomes

$$s_y = N\left(\frac{T_a}{A_a}\right) + \frac{N\, M_{LL+I}}{S_a}\left[\frac{1}{1 - (N\, T_a + N'\, T_t + N'\, T_w)\,\frac{u_{ae}}{M_{LL+I}}}\right] + \frac{N\, M_{DL}}{S_a} + \frac{N'\,(T_t + T_w)}{A_a} + \frac{N'\,(M_t + M_w)}{S_a} \qquad (16 \text{ revised})$$

and Eq. 17, also for the two-hinged arch, becomes

$$\frac{s_y}{N} = \frac{T_a}{A_a} + \left[\frac{M_{LL+I} + T_a\,\frac{e'}{N}}{S_a}\right]\left[\frac{1}{1 - (N\, T_a + N'\, T_t + N'\, T_w)\,\frac{u_{ae}}{M_{LL+I}}}\right] + \frac{M_{DL}}{S_a} + \frac{N'}{N}\left(\frac{T_t + T_w}{A_a} + \frac{M_t + M_w}{S_a}\right) \qquad (17 \text{ revised})$$

Similarly, for the hingeless rib, Eq. 21 becomes

$$\frac{s_y}{N} = \frac{T_a}{A_a} + \frac{M_{LL+I} + 0.7\, T_a\,\frac{e'}{N}}{S_a}\left[\frac{1}{1 - \left(\frac{N\, T_a + N'\, T_t + N'\, T_w}{T_a}\right)\left(\frac{\Delta m_{a1}}{M_{LL+I}}\right)}\right] + \frac{M_{DL}}{S_a} + \frac{N'}{N}\left[\frac{T_t + T_w}{A_a} + \frac{M_t + M_w}{S_a}\right] \qquad (21 \text{ revised})$$

In these revised equations the new term N' equals the load factor for temperature and wind loads (1.33 recommended). Present specifications account

for the improbability of the simultaneous occurrence of maximum dead load, live load, wind load, and temperature load by increasing the allowable total stress. The writers believe that a more rational approach, especially when the stress-load relation is nonlinear, is the use of load factors as in the revised Eqs. 16, 17, and 21.

Mr. Beskin states that conclusions 1 and 2 are incorrect. The arguments he has submitted do not uphold this statement. In stating that the factor $\frac{T\,u}{M}$ should be considered in design, Mr. Beskin agrees with conclusion 1 which states "The useful strength of an arch rib is dependent in part * * *" upon $T\,u$. The objections to conclusion 2 do not appear to be correct. The percentage error in total stress due to neglect of deflection effects is (approximately)

$$\text{Error } (\%) = E = 100\left[\frac{S_a + S_b\left(1 - \frac{S_a}{S_c}\right)}{S_a + S_b} - 1\right] \quad \ldots\ldots\ldots (126)$$

Differentiating this error with respect to S_a, the following expression results:

$$\frac{dE}{d(S_a)} = 100\left\{\frac{\frac{S_b}{S_c}\left[\frac{S_b + \frac{(S_a)^2}{S_c}}{\left(1 - \frac{S_a}{S_c}\right)^2}\right]}{(S_a + S_b)^2}\right\} \quad \ldots\ldots\ldots\ldots\ldots (127)$$

This result proves that $\frac{dE}{d(S_a)}$ cannot have a negative value, and that an increase in dead load for a given arch will always produce an increase in the error due to neglect of the deflection effects. In the hinged rib analyzed, an increase in dead load of 40% was found to increase the error in total stress by 10.8% (from 24.2% to 35%) in contrast to the 2% estimated by Mr. Beskin. Similarly, in the hingeless arch, a 40% increase in dead load caused a 1.3% (from 6.2% to 7.5%) increase in error as compared with the decrease predicted by Mr. Beskin.

Mr. Gronquist has suggested an interesting method of estimating deflection effects. The question of whether a correction factor should be applied to wind-load moments depends entirely on whether the critical wind loading produces a moment curve similar to that of the critical live load. The effect of wind loads on the rib will vary with the arrangement of the wind-bracing systems. In the Niagara design a symmetrical wind load was found to be critical for the quarter-point region and, therefore, the moments were not amplified by deflection.

Mr. Hirschthal has inquired about the considerations that determined the span length. The main abutments were located at a minimum safe height above the highest recorded ice level. The abutments so located determined the span length. With regard to his questions concerning the scope of the model tests made at Princeton University, it should be noted that these tests were not conducted as a part of the Niagara project, but as general research. The writers were aware of the work being done and, of course, were quite

interested in the results, since the work paralleled closely the design calculations under way at the time. Those in charge of the Princeton University tests expected to continue with corresponding tests on hingeless ribs, but this work became "a war casualty."

The selection of the hingeless type of rib was not as obvious a choice as Mr. Hirschthal infers. Preliminary discussions with experienced fabricating and erecting contractors indicated a strong preference for the two-hinged type. In addition, the preliminary designs by the usual "elastic theory" methods indicated that the total material saving in the hingeless arch would be small. The two-hinged arch could not logically be eliminated from consideration until an investigation of the effects of flexibility proved that a satisfactory design would require an excessive amount of material.

The use of three ribs would have been undesirable for the following reasons: (1) The shallower ribs resulting from a three-rib design would increase the objectionable flexibility effects; and (2) the unit stresses due to wind loading would be 50% to 100% greater in the lighter ribs of a three-rib design.

Esthetically and economically, the rib of constant depth is preferable for steel construction.

Mr. Hirschthal's assumption that the arch was designed for live load over half the span length has already been shown, in the third paragraph of this closure, to be incorrect. The live-load assumptions were more exacting than those that would be required by the specifications of the American Railway Engineering Association for Masonry Arches.

With reference to the design loading, the H20 loading was the standard for heavy traffic structures when the Rainbow arch was designed. The H20–S16 loading did not appear in the bridge specifications of the American Association of State Highway Officials until 1941.

The writers appreciate Mr. Stern's discussion, which presents an interesting method of attacking the problem of flexible arches.

ELMER K. TIMBY,[67] ASSOC. M. AM. SOC. C. E., LAWRENCE M. MEAD, JR.,[68] JUN. AM. SOC. C. E., AND ROBERT MCELDOWNEY, JR.,[69] ESQ.[69a]—Many valuable ideas have been presented in the discussions on the model studies of flexible two-hinged arches for which the writers are grateful. They were disappointed because of their inability to include in the investigation the behavior of flexible fixed-end arches, which several of the discussers would have liked. Although the model project was suggested by the design of the Rainbow Arch Bridge and served to confirm the designers' calculations on the two-hinged arch, it was felt that a more general investigation, covering the range of variables of rise to span, load, and stiffness, would be of more value to the profession than the study of only one combination of these variables. In the time available it was impossible to study both two-hinged and fixed-end arches. The fixed-arch program (and perhaps studies of parabolic arches) was planned for investiga-

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[69a] Received by the Secretary September 11, 1944.

tion in 1942, but World War II intervened, and no one was available for this work. It is hoped that when the war is over the project will be continued.

Mr. Ammann's suggestion that the "elastic theory" be called the "theory of the rigid system," and that the "deflection theory" be called the "theory of the flexible system" would clarify much confusion. Mr. Ammann misunderstood the writers' second conclusion, namely—that the test data would enable the designer to select a rib which in his (the designer's) opinion should be stiff enough to permit legitimate use of the "elastic theory."

Mr. Hirschthal questions the accuracy of the model studies and mentions the danger of applying them to so large a prototype. The writers maintain, however, that, if the accuracy of the measurements is increased in proportion to the decrease in the size of the model and the loads on the model, reasonable accuracy will not be sacrificed. For example, in the case of the model test which corresponded most closely to the proportions and loading of the Niagara arch, deflections could be measured to an accuracy of less than one part in fifty (or 2%) at the quarter point for maximum moment at the same point, even though the total deflection was only 0.07 in. on the model. An accuracy of 3% would probably be acceptable.

The writers cannot agree with Mr. Beskin that tests are of little value for analyzing or checking stress distributions of complicated or important structures. Mathematical treatment of many stress analysis problems is either inadequate or so complicated as to make test results far more practicable for actual design. As an aircraft stress analyst, Mr. Beskin must be familiar with the difficulty in treating, accurately, by mathematics, all phases of incomplete diagonal tension beams. For example, the method approved by the U. S. Army Air Corps for designing open section web stiffeners is based on an empirical formula developed to fit the results of a large number of static tests. Although several theoretical solutions of the problem appear reasonable and seem to cover the more important variables, none actually was verified sufficiently by test to be usable without unwarranted conservatisms.

In the case of a structure as important and costly as the Rainbow Arch Bridge, the model tests would seem justified for checking purposes even if they only proved (as they did) that the theoretical treatment included enough variables and was sufficiently accurate to insure a safe design.

In regard to the variation of dead load across the span, the writers felt that in such a general investigation, and with the limited time available, it was better to select a distribution that produced no dead-load moment—a condition sought in practical design—rather than to select some arbitrary percentage variation across the span. Undoubtedly the particular variation chosen will have some effect on the amplified final moments, especially in the case of the large ratios of dead load to stiffness and of rise to span. The actual dead-load distribution used varied as $\frac{r^2}{y^3}$ which reduces to $\frac{1}{r \cos^3}$, as Mr. Beskin deduced.

Mr. Beskin is to be congratulated for developing a theoretical formula which checks the test results so accurately. The writers consider their work well justified if it has stimulated the development of such theoretical solutions and has served as a check for them.

JOHN I. COPP,[70] ASSOC. M. AM. SOC. C. E., AND KARL DE VRIES,[71] WILLIAM H. JAMESON,[72] AND JONATHAN JONES,[73] MEMBERS, AM. SOC. C. E.[73a]—The writers appreciate that the greater portion of the study given by many members of the Society to the papers of this Symposium was given to the first paper, on design, since that paper is naturally of more direct interest to designing engineers than is one covering fabrication and erection controls.

The method of weighing the crown thrust at closure has also been commented on, and a request has been made for a more complete description of the "weighing capsules." The writers would prefer to refer to the conclusion of the paper, and to emphasize that, in their opinion, weighing of the crown thrust of a properly constructed arch is unnecessary, and on an improperly or carelessly constructed arch it is useless.

Corrections for *Transactions:* In October, 1943, *Proceedings*, page 1226, change the sentence in lines 4 to 6 to read: "The method here used, based on the physical concept of the elastic centroid defined in the paper, is taken from 'Die graphische Statik der Baukonstruktionen,' by H. Müller-Breslau[17]"; in footnote 17 on page 1226, change "Vol. 2, Pt. 2, 1925" to "Vol. 2, Pt. 2, 1925 Edition."

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[73a] Received by the Secretary December 18, 1944.